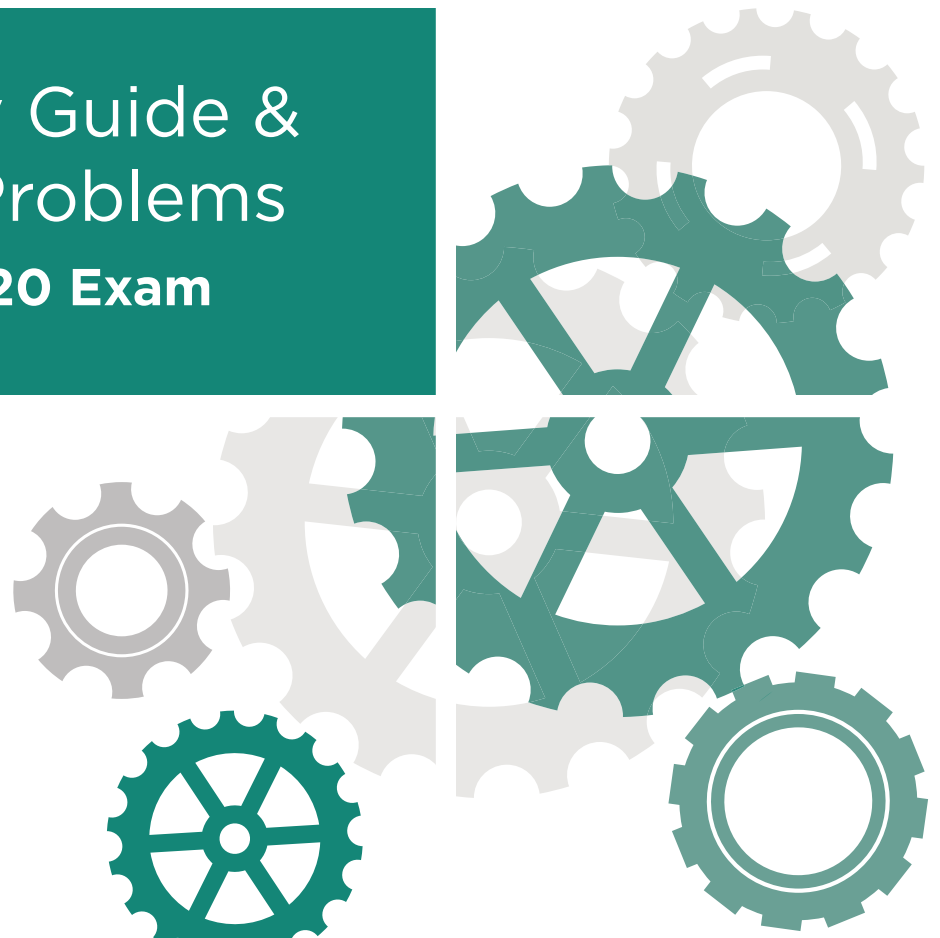


Textbook & Full Exam

Mechanical
FE

Technical Study Guide &
250+ Practice Problems
Updated for July 2020 Exam



Learn the key concepts and skills necessary to pass the FE Exam



Engineering
Pro Guides

by Justin Kauwale, P.E.

Mechanical FE Exam: Textbook & Full Exam

by Justin Kauwale, PE

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Mechanical FE Textbook & Full Exam

How to pass the Mechanical FE exam

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0 - Introduction

How to Study for and Pass the FE Exam



Section 0.0 - Introduction

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1.0 INTRODUCTION

One of the most important steps in an engineer's career is obtaining the professional engineering (P.E.) license. It allows an individual to legally practice engineering in the state of licensure. This credential can also help to obtain higher compensation and develop a credible reputation. The first step towards obtaining your P.E. is passing the Fundamentals of Engineering (F.E.) Exam. Both tests are administered by the National Council of Examiners for Engineering and Surveying (NCEES). The FE Exam is a year round computer based test that can be taken as early as your senior year in college or with at least 3 years of engineering-related work experience. Once passed, the FE Exam will certify you as an Engineering in Training (EIT). With enough experience after passing the EIT, you will become eligible for the PE Exam. Engineering Pro Guides focuses on helping engineers pass the NCEES exam through the use of free content on the website, <http://www.engproguides.com> and through the creation of books like sample exams and guides that outline how to pass the PE exam.

In the FE exam you will not be able to bring in any outside reference material. You will be given the *NCEES FE Exam Reference Handbook*, which contains all the necessary equations, tables, and graphs that you will need to solve each problem. The *NCEES FE Exam Reference Handbook* will be provided as a searchable electronic pdf during the test. The key to passing the FE exam is understanding the key concepts and skills that are tested on the exam and becoming familiar with using this handbook to solve each problems in approximately 2-3 minutes. Although the NCEES handbook provides the necessary equations for the exam, knowing how to apply them and which equations to use requires an understanding of the concepts and practice of the skills. The FE Exam is available for 6 disciplines plus a generic engineering discipline. This technical guide teaches you the key concepts and skills required to pass the Mechanical F.E. Exam in a single document.

1.1 EXAM FORMAT

How is the exam formatted?

The FE exam format and additional exam day information can be found on the NCEES Examinee Guide (<https://ncees.org/exams/examinee-guide/>). The entire exam period is about 6 hours, with 2 minutes for signing agreements, 8 minutes for tutorials, and one break up to 25 minutes. You will have a total of 5 hours 20 minutes of actual exam time to solve 110 problems, which equates to about 2.9 minutes per problem if spread out evenly. The test is broken up into two sessions. The length of each session is determined by the number of problems, 55 problems per session, and not the time. So, you could spend more or less than half the time on the first session, and the remaining 5 hours 20 minutes will be allotted for the second session. Since the first session doesn't have a halfway time limit, it is very important to keep watch of the clock to make sure you have enough time for the second session. Before each session is completed, you are allowed to go back to problems that you may have skipped or want to check in that session. However, once the first session is completed and submitted, you are no longer allowed to revisit the questions in that session. There is a 25 minute break in between the sessions. You are allowed to take less than a 25 minute break or no break at all, but this does not increase the time you have to answer

exam questions. No points are deducted for incorrect answers, so be sure to provide an answer for all questions, even if it is a guess. The final results are scaled based on the exam difficulty. There are five types of question formats that could be presented on the exam.

1. Multiple Choice (4 choices) – Select one option, *majority of questions in the exam*
2. Multiple Answers – Select multiple answers that are correct
3. Select by Clicking – Click on a point on a graph, etc
4. Drag and Drop – Matching, sorting, labeling, etc
5. Fill in the Blank – Type in the answer

The types of questions and number of questions per topic will be based on the outline provided by NCEES, discussed in the next section. These topics will not be labeled on the test. Finally, the NCEES Examinee Guide states that there will be some questions that will not be scored in the exam. These are questions that are tested for their quality and possible use in future exams. Your final results will be given to you 7-10 days after you take the exam.

1.2 KEY CONCEPTS AND SKILLS

How are the key concepts and skills determined?

The key concepts and skills tested in the sample exams and taught in this technical study guide were first developed through an analysis of the topics and information presented by NCEES. The above factors related to timing is considered. The Mechanical FE exam will focus on the following topics as indicated by NCEES. (<https://ncees.org/engineering/fe/>):

- 1 Mathematics - (6-9 questions)**
 - i) *Analytic Geometry*
 - ii) *Calculus*
 - iii) *Ordinary Differential Equations*
 - iv) *Linear Algebra*
 - v) *Numerical Methods*
 - vi) *Algorithm and Logic Development*
- 2 Probability and Statistics - (4-6 questions)**
 - i) *Probability Distributions*
 - ii) *Measures of Central Tendencies and Dispersions*
 - iii) *Expected Value*
 - iv) *Regression, Curve Fitting, Goodness of fit*
- 3 Ethics and Professional Practice - (4-6 questions)**
 - i) *Codes of Ethics*
 - ii) *Public Health, Safety, and Welfare*
 - iii) *Intellectual Property*
 - iv) *Societal Considerations*
- 4 Engineering Economics - (4-6 questions)**
 - i) *Time Value of Money*
 - ii) *Cost: incremental, average, sunk, estimating*
 - iii) *Economic Analyses*

5 Electricity and Magnetism (5-8 questions)

- i) *Electrical Fundamentals*
- ii) *DC Circuit Analysis*
- iii) *AC Circuit Analysis*
- iv) *Motors and Generators*

6 Statics (9-14 questions)

- i) *Resultants of Force Systems*
- ii) *Concurrent Force Systems*
- iii) *Equilibrium of Rigid Bodies*
- iv) *Frames and Trusses*
- v) *Centroids and Moments of Inertia*
- vi) *Static friction*

7 Dynamics, Kinematics, and Vibrations (10-15 questions)

- i) *Kinematics of Particles*
- ii) *Kinetic Friction*
- iii) *Newton's Second Law for Particles*
- iv) *Work-Energy of Particles*
- v) *Impulse-Momentum of Particles*
- vi) *Kinematics of Rigid Bodies*
- vii) *Kinematics of Mechanisms*
- viii) *Newton's Second Law for Rigid Bodies*
- ix) *Work-Energy of Rigid Bodies*
- x) *Impulse-Momentum of Rigid Bodies*
- xi) *Free and Forced Vibrations*

8 Mechanics of Materials (9-14 questions)

- i) *Shear and Moment Diagrams*
- ii) *Stress Transformations and Mohr's Circle*
- iii) *Stress and Strain caused by Axial Loads*
- iv) *Stress and Strain caused by Bending Loads*
- v) *Stress and Strain caused by Torsion*
- vi) *Stress and Strain caused by Shear*
- vii) *Stress and Strain caused by Temperature Changes*
- viii) *Combined Loading*
- ix) *Deformations*
- x) *Column Buckling*
- xi) *Statically Indeterminate Systems*

9 Material Properties and Processing (7-11 questions)

- i) *Properties: chemical, electrical, mechanical, physical, thermal*
- ii) *Stress-Strain Diagrams*
- iii) *Ferrous Metals*
- iv) *Nonferrous Metals*
- v) *Engineered Materials*
- vi) *Manufacturing Processes*
- vii) *Phase Diagrams, Phase Transformation, and Heat Treating*
- viii) *Materials Selection*
- ix) *Corrosion Mechanisms and Control*



- x) *Failure Mechanisms*
- 10 Fluid Mechanics (10-15 questions)**
 - i) *Fluid Properties*
 - ii) *Fluid Statics*
 - iii) *Energy, Impulse, Momentum*
 - iv) *Internal Flow*
 - v) *External Flow*
 - vi) *Compressible Flow*
 - vii) *Power and Efficiency*
 - viii) *Performance Curves*
 - ix) *Scaling Laws for Fans, Pumps, and Compressors*
- 11 Thermodynamics (10-15 questions)**
 - i) *Properties of Ideal Gases and Pure Substances*
 - ii) *Energy Transfers*
 - iii) *Laws of Thermodynamics*
 - iv) *Processes*
 - v) *Performance of Components*
 - vi) *Power Cycles*
 - vii) *Refrigeration and Heat Pump Cycles*
 - viii) *Non-reacting Mixtures of Gases*
 - ix) *Psychrometrics*
 - x) *Heating, Ventilation, and Air Conditioning (HVAC) processes*
 - xi) *Combustion and Combustion Products*
- 12 Heat Transfer (7-11 questions)**
 - i) *Conduction*
 - ii) *Convection*
 - iii) *Radiation*
 - iv) *Transient Processes*
 - v) *Heat Exchangers*
- 13 Measurement, Instrumentation, and Controls (5-8 questions)**
 - i) *Sensors*
 - ii) *Control Diagrams*
 - iii) *Dynamic System Response*
 - iv) *Measurement Uncertainty*
- 14 Mechanical Design and Analysis (10-15 questions)**
 - i) *Stress Analysis of Machine Elements*
 - ii) *Failure Theories and Analysis*
 - iii) *Deformation and Stiffness*
 - iv) *Springs*
 - v) *Pressure Vessels and Piping*
 - vi) *Bearings*
 - vii) *Power Screws*
 - viii) *Power Transmission*
 - ix) *Joining Methods*
 - x) *Manufacturability*
 - xi) *Quality and Reliability*

xii) *Components (Hydraulic, Pneumatic, Electrochemical)*

xiii) *Engineering Drawing Interpretations and Geometric Dimensioning and Tolerancing*

Each of these broad topics were investigated and filtered for concepts and skills that met the following criteria:

(1) First, the concept and skill must be fundamental principles taught in college. The test is intended for the engineer right out of college without work or practical experience. The exam will focus on fundamental engineering principles you will need during your career. However, since the Mechanical Engineering discipline is broad, the exam will be based on the general knowledge that each Mechanical Engineer develops in school and will not include an in-depth, higher level analysis of a specific topic. The subjects listed above are the basic curriculum that Mechanical Engineers should encounter before they graduate.

(2) Second, the skill and concept must be testable in roughly 2.9 minutes per problem. There are (110) questions on the Mechanical FE exam and you will be provided with 5 hours 20 minutes to complete the exam. This results in an average of 2.9 minutes per problem. This criterion limits the complexity of the exam problems and the resulting solutions. For example, pressure drop calculations are common in Fluids, but the calculation is often very lengthy because of the number of steps involved, especially if a unique fluid and flow condition is used. Thus, common fluids like water/air and common pipe/duct materials are used.

(3) Third, the information and equations required to solve the problems should be in the *NCEES FE Reference Handbook*. Since you are not allowed to bring in outside resource, the Handbook and along with any information given to you in the problem should provide you with sufficient information needed to solve the problems. It is extremely unlikely that you will need an equation that is not given to you in the reference handbook. Thus, the handbook is an additional resource for understanding the types of questions that could be asked. Note that the *NCEES FE Reference Handbook* contains extraneous information for the Mechanical FE exam, since the same resource is used across all tested disciplines. To narrow down the relevant topics, the handbook was cross referenced with the NCEES Mechanical FE outline mentioned in the last section. Lastly, the solution may still require a variation of the equations in the reference handbook. Therefore it is very important to understand how to use these equations, as well as the variables and the units that the equations require.

(4) The F.E. Exam tests the background engineering concepts and skills for a practicing Mechanical Engineer and not the derivation of the topic or concept. The exam is intended to prove that the test taker is minimally competent to practice as an engineer in training and has the basic understanding of Mechanical Engineering principles. This background knowledge is necessary for the practicing engineer to understand how engineering concepts and skills are applied in the field. Therefore, the exam is less concerned with theory and more with how these concepts and skills can be applied. For example, the F.E. exam is less interested with the derivation of angular momentum equations and more with how to solve for resultant forces or final velocity conditions.

In summary, this book is intended to teach the necessary skills and concepts to develop a minimally competent, practicing Mechanical Engineer in Training, capable of passing the F.E. exam. This book and the sample exam do this through the following means:

- (1) Teaching common skills, principles, and concepts in the Mechanical field.**
- (2) Providing sample problems that can be completed in roughly 2-3 minutes per problem.**
- (3) Teaching how to use and apply the equations in the *NCEES FE Reference Handbook*.**
- (4) Teaching the application of the skill and concept for an engineer in training.**

1.3 UNITS

The units that are used in the F.E. Exam are the International System of Units (SI) and the United States Customary System Units (USCS). The equations in the *NCEES FE Reference Handbook* are more generic and does not necessarily differentiate between SI or USCS units. Therefore, it is very important, especially with the USCS problems, to make sure all necessary conversion factors are used and that the units cancel out to the unit of your desired final answer. Be aware of the use of the g_c conversion factor in USCS problems. See the fluids section for an explanation of the g_c term.

2.0 DISCLAIMER

In no event will Engineering Pro Guides be liable for any incidental, indirect, consequential, punitive or special damages of any kind, or any other damages whatsoever, including, without limitation, those resulting from loss of profit, loss of contracts, loss of reputation, goodwill, data, information, income, anticipated savings or business relationships, whether or not Engineering Pro Guides has been advised of the possibility of such damage, arising out of or in connection with the use of this document or any referenced documents and/or websites.

This book was created on the basis of determining an independent interpretation of the minimum required knowledge and skills of an engineer in training. In no way does this document represent the National Council of Examiners for Engineers and Surveying views or the views of any other professional engineering society.

3.0 HOW TO USE THIS BOOK

This book is organized into the topics as designated by the NCEES. These topics include:

- Section 0.0: Introduction
- Section 1.0: Mathematics
- Section 2.0: Probability and Statistics
- Section 3.0: Ethics and Professional Practice
- Section 4.0: Engineering Economics

- Section 5.0: Electricity and Magnetism
- Section 6.0: Statics
- Section 7.0: Dynamics, Kinematics, and Vibrations
- Section 8.0: Mechanics of Materials
- Section 9.0: Material Properties and Processing
- Section 10.0: Fluid Mechanics
- Section 11.0: Thermodynamics
- Section 12.0: Heat Transfer
- Section 13.0: Measurement, Instrumentation, and Controls
- Section 14.0: Mechanical Design and Analysis

First, it is recommended that the engineer in training gather the *NCEES FE Reference Handbook*.

Second, proceed through the book in the order designated. Go through and first read the material of the section, then complete the practice problems designated for that section. If you have trouble with the practice problems, review the material and then read the solutions. These problems are meant to practice the application of the skill or concept presented in the section. The problems are exam difficulty level.

Following the completion of each of the sections, it is recommended that you go through the checklists presented on the Engineering Pro Guides website. These checklists pose vital questions to the engineer in training about their understanding of all the skills and concepts presented in this book. If you are not confident with any of the items, please go back and revisit the section.

Finally, set aside a five-hour twenty-minute block of uninterrupted time to complete a sample exam. Gather your references and calculator and create a test-like environment. Set a timer and proceed to take the sample exam, which can be purchased separately. Remember that the exam is only 55 problems each for the first and second sessions and does not encompass all the possible items that can appear on an exam, but it should give you an idea of your level of readiness for the exam.

4.0 PRACTICE EXAM TIPS

Engineering Pro Guides practice exam problems can be used in multiple ways, depending on where you are in your study process. If you are at the beginning or middle, it can be used to test your competency, gain an understanding and feel for the test format, and help to highlight target areas to study. If you are at the end, it can be used to determine your preparedness for the real exam. Remember that the questions are a sample of the many topics that may be tested and are limited to fit a full exam length and therefore is not comprehensive of all concepts. Also the practice exam problems are split up throughout the entire book.

Because the exam is written to be similar to the difficulty and format of the NCEES exam, it is recommended that the test be completed in one sitting and timed for two hours forty minutes to simulate half of the real exam. This will give you a better indication of your status of preparation

for the exam. If you are at the ending of your studying, it is recommended to couple this exam with the second section to simulate the full exam test day.

Review the exam day rules and replicate the environment for the real test as much as possible, including the type of calculator you may use and the acceptable references. Keep a watch or clock next to you to gauge your pace for 55 questions in 2 hours 40 minutes.

Based on the NCEES website, the following are general rules for exam day.

Allowed:

1. ID used for admission
2. Approved calculator (2 recommended for backup. The backup will be stored with your personal items)
3. Eyeglasses
4. Light sweater or jacket
5. Test center locker key
6. Test center provided booklet and marker
7. Test center comfort aids, approved upon visual inspection. See the *Pearson VUE Comfort Aid List* on the Pearson VUE website (includes medicine – inhaler, aspirin not in bottle, eye drops, cough drops, etc and mobility devices – crutches, wheelchairs, etc. Tissues and earplugs must be provided by the test center.)
8. Religious head coverings

Prohibited:

1. Cell phones
2. Electronic Devices (other than approved calculator)
3. Watches
4. Wallets and Purses
5. Hats and hoods
6. Bags
7. Coats
8. Books
9. Pens, Pencils, Erasers
10. Food, Drinks
11. Weapons
12. Tobacco
13. Eyeglass cases
14. Scratch Paper (all writing devices are provided)

Most test centers will have lockers for you to store your personal items. For additional references on exam day policies, exam day processes, and items to bring on your exam day, review the NCEES Examinee Guide:

<http://ncees.org/exams/examinee-guide/>

For best use of your time, answer the questions that you know first and return to the questions that you are unfamiliar with later. On the computer based test, you are able to bookmark the answers you may want to come back to later. Once all the known questions are answered, go through the test again and attempt to answer the remaining questions by level of difficulty. If time allots, review your answers.

If you are stuck on a question, seek the following avenues.

1. *NCEES FE Reference Handbook*: It is important to understand the *NCEES FE Reference Handbook*. During times of uncertainty, this will likely lead you to your answers. Determine the key concept that is being asked in the question and refer to this reference book. Remember that the reference is searchable, so you will be able to do a search by keyword (Ctrl+F). Additional tips on this resource is discussed in the next section.
2. *Process of Elimination*: In most questions, there are only four possible choices for each question. Ask yourself if there is an answer that does not make sense and eliminate it. Further narrow down the answer that are derived from equations or concepts that you know are not right and are instead meant to deceive the test taker. See if there are answers that are similar or separated by something like a conversion error. This may be an indication that the correct equation was used.
3. *Educated Guess*: Remember that there is no penalty for wrong answers. Hopefully with the process of elimination you are able to narrow down as many answers as possible and are able to create an educated guess.
4. If the time is almost up and there are still unanswered questions remaining, determine whether it makes sense to check for mistakes on the problems you do know how to solve, or to tackle the unanswered problems.

Typical Exam Verbiage/Design:

1. *Most Nearly*: Due to rounding differences, the exam answers may not match yours exactly and in fact may not even closely resemble your answer. NCEES uses the term “most nearly” to test your confidence in your solution. When the question prompts you with “most nearly”, choose the answer that most closely matches yours, whether it be greater than or lesser to your value.
2. *Irrelevant Information*: The exam is intended to test your overall understanding of concepts. At times the question will include unnecessary information that is meant to misdirect you.
3. *Deceiving Answers*: NCEES wants to know that you are able to determine the appropriate methods for the solutions. There are answers that were intentionally produced from wrong equations to mislead the test taker. For example, you may forget

a $1/2$ in the formula, $KE = (1/2)MV^2$ and there would be two answers each off by a factor of $1/2$.

4. Do Not Overanalyze: The exam questions are meant to be completed in less than 3 minutes. Therefore, they are intended to be written as straight forward as possible. Do not be tempted to overanalyze the meaning of a question. This will only lead you down the wrong path.

Review the Solutions:

Once the sample test is completed, grade your results. Measure your aptitude in speed, concept comprehension, and overall score. If your score is above the 75% range then you are in good shape. This 75% score is only applicable if you have prepared completely for the exam. If you are just starting out, then do not be worried about a low score. This number is also just a range; there is no finite score to determine passing the test. Instead, NCEES calibrates the results against practicing professional engineers. See this page <http://ncees.org/exams/scoring-process/> for a better understanding of how NCEES grades the scores.

Review the answers that you got wrong and use the solutions as a learning tool on how to address these types of problems. Compare the types of questions you are missing with the NCEES outline of topics and determine where you should focus your studying. Finally repeat as many practice problems as you can to get a better grasp of the test and to continually improve your score.

5.0 NCEES FE REFERENCE HANDBOOK

As previously mentioned, the *NCEES FE Reference Handbook* is the only reference material you will have during the exam. Therefore, it is very important to use this reference book when doing practice problems. You should become familiar with the layout of the book, how to apply the equations, what the variables mean, what units the equations are in, and where to find common constants, tables, and graphs. The *NCEES FE Reference Handbook* can be purchased as a hardcopy on the NCEES website or downloaded a free pdf of the latest version from your *MyNCEES* account. I would recommend studying from the pdf to become familiar with using the reference book electronically with the search (Ctrl+F) options. The index will not be provided during the real exam. When studying, notice how the Handbook is organized and how it is broken out by subject, then by discipline. Take some time go browse through the entire reference handbook to see where different equations are located. Realize that some of the Mechanical FE questions may overlap with other disciplines, like Civil and Electrical.

5.1 UNIT CONVERSION

The first section of the *NCEES FE Reference Handbook* has a list of typical unit conversions as well as common constants, such as the universal gas constant, gravity, Stefan-Boltzmann constant.

5.2 TABLES AND GRAPHS

It is important to be able to quickly navigate through the *NCEES FE Reference Handbook* and know where the common tables are used across multiple subjects.

The following are examples of common tables or graphs that you should be aware of.

- Area Moment of Inertia & Centroid – *Statics*
- Mass Moment of Inertia & Centroid – *Dynamics*
- Modulus of Elasticity – *Material Properties*
- Metal Densities – *Material Properties*
- Coefficient of Thermal Expansion – *Material Properties*
- Beam Deflection – *Mechanics of Materials*
- Steam Tables – *Thermodynamics*
- Specific Heat Capacities – *Thermodynamics*
- Water Properties – *Fluid Mechanics*
- Moody Diagram/Roughness Coefficients – *Fluid Mechanics*
- Drag Coefficients – *Fluid Mechanics*
- Hazen-Williams Coefficients – *Civil Engineering*
- Beam Analysis (additional) – *Civil Engineering*
- Buckling – *Civil Engineering*
- Mechanical Application – *Mechanical Engineering*

6.0 PAST EXAMS

6.1 PASS RATES ON SURVEY VS. NCEES

The NCEES website indicates that 77% of Mechanical FE test takers pass the exam. These pass rates only include first time test takers that have attended an accredited engineering program and took the test within 12 months of graduation.

6.2 ESTIMATED CUT SCORE

Since the exam is weighted, the cut score is not clearly defined and is never posted by NCEES. The general online consensus is that the passing rate is about 50-55% correct. Our goal with this book is to get you to a score of at least 70%. You should be able to obtain at least this amount to increase your confidence of passing.

1 - Mathematics



Section 1.0 – Mathematics

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1.0 INTRODUCTION

Mathematics accounts for approximately 6 to 9 questions on the Mechanical FE exam. The topics covered in this section include Analytic Geometry, Calculus, Ordinary Differential Equations, Linear Algebra, Numerical Methods, and Algorithm and Logic Development. At first glance, these topics may seem broad and daunting, but it can easily be narrowed down to the *NCEES FE Reference Handbook*, as they relate to Mechanical Engineering. Although a lot is covered in this chapter, keep in mind that mathematics will only account for about 8% of the exam and only two minutes are allotted per problem. Mathematics is a foundation for the other mechanical engineering topics, so unless you have been away from school for very long, you should be able to breeze through this chapter.

Section 1.0 Mathematics (6 to 9 Problems)		
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	Section 1.0	Introduction
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1B	Section 3.0	Calculus
1C	Section 4.0	Ordinary Differential Equations
1D	Section 5.0	Linear Algebra
1E	Section 6.0	Numerical Methods
1F	Section 7.0	Algorithm and Logic Development
	Section 8.0	Practice Exam Problems

2.0 ANALYTIC GEOMETRY

Analytic geometry uses algebra to characterize various geometric objects such as shapes, lines and points.

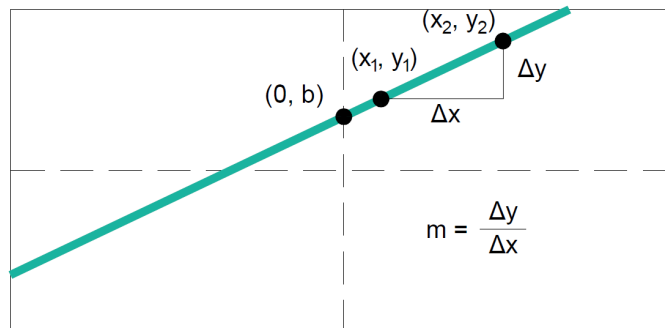


Figure 1: The slope of a line can be found with the difference between the y-values and x-values of two points.

2.1 FIND LINE EQUATION GIVEN TWO POINTS

Given two points, a line connecting the two points can be found with the process below.

First solve for slope, “m”, which is equal to the change in y over the change in x.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Next, solve for the y-intercept, “b”, using the following equation for a line. First, plug in the value of the slope, “m”, and one of the (x, y) points along the line. Then, solve for the y-intercept, “b”.

$$y - \text{intercept equation for line} \rightarrow y = mx + b$$

$$y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) * x_1 + b$$

Finally, replace “b” in the equation and you will have found the equation of the line.

2.2 PARALLEL AND PERPENDICULAR LINES

Another important skill is being able to calculate the equation for lines that are parallel or perpendicular to each other.

Parallel lines share the same slope.

$$\text{Parallel Lines} \rightarrow m_1 = m_2$$

The equation of a parallel line can then be determined by finding the vertical offset between the two parallel lines. Add the offset to the y-intercept of one line to find the equation of the other.

$$\text{Line 1} \rightarrow y = m_1x + b$$

$$\text{Line 2} \rightarrow y = m_2x + (b + \text{offset}); \text{ where } m_1 = m_2$$

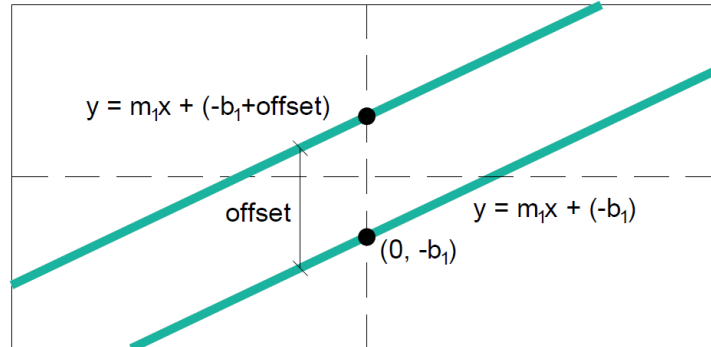


Figure 2: Parallel lines have the same slope and never intersect.

Perpendicular lines have inverse, negative slopes.

$$\text{Perpendicular Lines} \rightarrow -\frac{1}{m_1} = m_2$$

Find the slope of the perpendicular line, then solve for the new y-intercept, “b₂”, by substituting one of the (x, y) coordinates on the line.

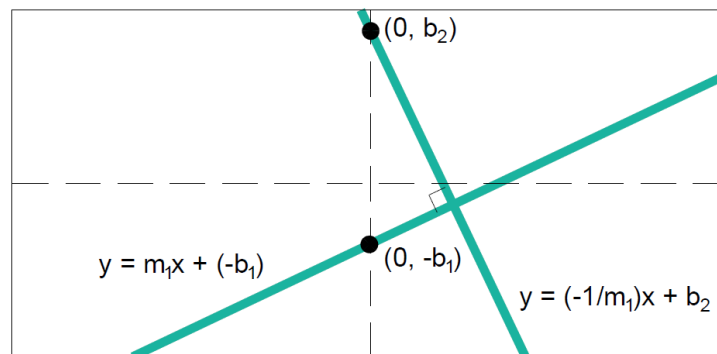


Figure 3: This figure shows the perpendicular intersection of two lines.

2.3 DISTANCE BETWEEN TWO POINTS

The distance between two points can be calculated with the Pythagorean Theorem.

2 – Probability & Statistics



Section 2.0 – Probability and Statistics

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1.0 INTRODUCTION

Probability and Statistics accounts for approximately 4 to 6 questions on the Mechanical FE exam. Statistics is primarily used in Machine Design for statistical quality control, which is covered under Section 14.0 Mechanical Design and Analysis, under the topics Quality & Reliability. This section focuses on the following NCEES Outline topics, Probability Distributions and Regression Curve Fitting.

Probability Distribution involves applying a mathematical formula to describe the probability of a measured variable occurring at a certain value. This is useful for characterizing the measured output of any mechanical system property when you are taking a sample of a larger number. For example, you measure the weight of 100 products, but this is only a sample of the 10,000 products that are produced. A probability distribution will help to characterize all 10,000 products.

Regression curve fitting involves measuring a variable as a function of another variable, then plotting the data points and assigning a mathematical formula to approximate the function. This is useful in predicting how a change in one variable will affect another.

Section 2.0 Probability and Statistics (4 to 6 Problems)		
NCEES Outline Value	Engineering Pro Guides	
		Section 1.0
2B	Section 2.0	Measures of central tendencies and dispersions (e.g., mean, mode, standard deviation, confidence intervals)
2A	Section 3.0	Probability Distributions (e.g., normal, binomial, empirical, discrete, continuous)
2C	Section 4.0	Expected value (weighted average) in decision making
2D	Section 5.0	Regression (linear, multiple), curve fitting, and goodness of fit (e.g., correlation coefficient, least squares)
	Section 6.0	Practice Exam Problems

2.0 MEASURES OF CENTRAL TENDENCIES AND DISPERSIONS

Before you get to probability distributions, you need to understand some of the basic topics in probability like the difference between samples and population, mean, mode, standard deviation. As you go through these topics, you should remember that probability is used in mechanical engineering to measure the reliability of a set of data points.

2.1 MEAN OR AVERAGE

The mean of a set of data points is calculated by summing up all the values and dividing by the total number of data points. The mean is also known as the average.

$$\bar{x} = \frac{\sum_1^n x_i}{n}$$

n = number of data points; x_i = measured value; \bar{x} = mean or average

Sometimes this term can also be called arithmetic mean. An example calculation for finding the mean is shown below for the sample data set.

Data set → {1,2,2,2,3,4,10}

$$\bar{x} = \frac{1 + 2 + 2 + 2 + 3 + 4 + 10}{7} = 3.43$$

2.2 MODE

The mode is the measured value that appears the most in a set of data points. In the sample data set from the previous topic, the mode will be “2”.

2.3 MEDIAN

The median is the measured value that occurs in the middle of the data set. The median is found by first ordering all the data points in ascending or descending order, then finding the middle value. If there is no middle value (i.e. there are an even number of data points), then you must take the average between the two middle values.

Data set → {1,2,2, 2, 3,4,10}

The median in the previous example is “2”.

2.4 GEOMETRIC MEAN

The geometric mean is used to give equal weight to a set of data points with high volatility. The geometric mean is found by multiplying the data points and taking the nth root of the product, where n is equal to the number of data points.

distribution formulas, (1) Binomial, (2) Normal, (3) t-Distribution and (4) χ^2 Distribution. These tables generally describe the probability of obtaining “x” successes, within “n” attempts or samples, with an individual probability of success equal to “P”. This is often used to predict and test the probability of success, given a number of samples.

3.1 BINOMIAL DISTRIBUTION

A binomial distribution is used when there are only 2 outcomes. Typically in mechanical engineering, these two outcomes are described as success or failure, pass or do not pass, and defective or satisfactory. The following table shows the *cumulative* binomial probability of success, given “n” independent trials or experiments, “x” successes and “p” individual probability. Say for example you are to flip a coin. The amount of times you flip a coin is “n”, the number of trials. These trials are independent because one does not affect the other. Let’s assume that heads is a success and tails is a failure. The number of heads flipped throughout the trials is “x” the number of successes. Finally, the likelihood of each flip being a heads is 50%, so “p” the individual probability is 0.5. For a set number of trials, n, the binomial distribution will graph out the probabilities of x successes from 0 to n trials. The cumulative binominal probability table includes all probabilities of success from 0 to x. The main objective is to understand how to read and use the Cumulative Binomial Distribution table.

Table 1: This table is read by first identifying the correct column to look at. The columns corresponds to the probability of success for each individual trial or sample. For example, you can assume the individual probability of success is 0.1, meaning that the probability that each product will come out successful is 10%. Next, determine the combination that you want. A combination is the number of successes “x” out of a total number of trials “n”. For example, if you want to know the probability of 0 successes out of 2 trials, then you would find the probability as 0.81. This means that there is an 81% chance that you will select two samples or run two trials and both will be failures, meaning you have zero successes. This table is shown as cumulative, so if you navigate to right below that point you will see a value of 0.99. This means that there is a 99% probability that out of two trials you will either have 0 successes or 1 success. If you calculate the difference between $x=1$ and $x=0$, then you will have the probability that there is exactly 1 success, which is $0.99-0.81=0.18$ or 18%.

Cumulative Binomial Distribution

		0.1	0.2	0.3	0.4	0.5
n=2	x=0	0.81	0.64	0.49	0.36	0.25
	1	0.99	0.96	0.91	0.84	0.75
	2					

It is very important that you understand the probability of success is cumulative in these tables as you move from $x=0$ down to $x=1$.

3.2 NORMAL DISTRIBUTION

A normal distribution is shown as the figure below. In order to answer these problems, you have to use a different method from the binomial distribution table. The outline of the normal distribution curve may look similar to the binomial distribution at times, but its values are different.

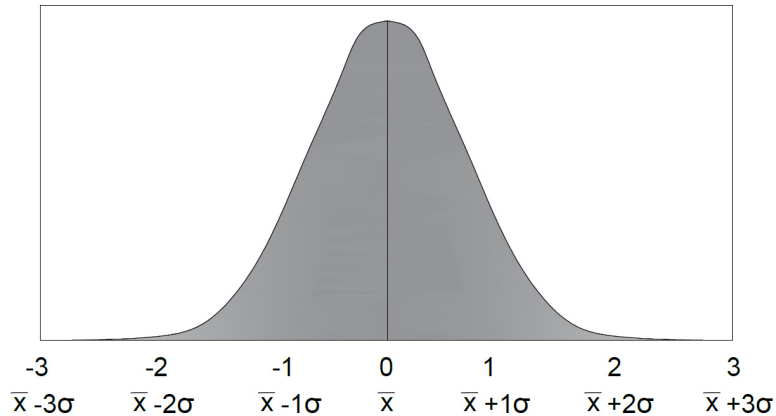


Figure 1: This figure shows the normal or Gaussian distribution. This can also be shown in table form. It shows the probability of a sample's value appearing a certain standard deviation away from the mean. There is a high probability of the value appearing at the mean, then 1 standard deviation away, then less probability at 2 deviations away and finally least probability at 3 standard deviations away.

This graph can be used as follows. The first way is through the $R(x)$ values. These values tell you the probability that the sample will occur in the range to the right of the (x) value. The (x) value is the number of standard deviations away from the arithmetic mean. This tells you the probability that the sample will occur in the range right of the (x) value of standard deviations away from the arithmetic mean.

3 – Ethics & Professional Practice



Section 3.0 – Ethics and Professional Practice

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1.0 INTRODUCTION

Ethics and Professional Practice accounts for approximately 4 to 6 questions on the Mechanical FE exam. The NCEES outline provides the following topics on its outline, Codes of Ethics, Agreements and Contracts, Ethical and Legal Considerations, Professional Liability, Public Health, Safety and Welfare. There is no readily available or commonly used content that covers these topics, except for what is shown on the NCEES website and the *NCEES FE Reference Handbook*. In addition, the topics cover concepts that are open for interpretation. These two facts make it very difficult to fairly test this topic.



Section 3.0 Ethics and Professional Practice (4 to 6 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
3A	Section 1.1	Codes of ethics (e.g., NCEES Model Law, professional and technical societies, ethical and legal considerations)
3B	Section 1.2	Public health, safety, and welfare
3C	Section 1.3	Intellectual property (e.g., copyright, trade secrets, patents, trademarks)
3D	Section 1.4	Societal considerations (e.g., economic, sustainability, life-cycle analysis, environmental)
	Section 2.0	Practice Exam Problems

1.1 CODES OF ETHICS

There are a couple of things you can do to prepare for questions on ethics.

(1) The first can be found in the *NCEES FE Reference Handbook*. There are about 8 pages on Ethics. This section covers the Codes of Ethics, Intellectual Property and the NCEES Model Law and Model Rules.

(2) The second is to read through the Model Law. The model law is published by NCEES and it is on the NCEES website. The link is shown below. The model law has very general regulations to govern engineering for the purposes of keeping the public safe. It is important to note that these laws are only ideas and it is up to each authority having jurisdiction like your State board to come up with their own laws.

NCEES Publications Website: <https://ncees.org/about/publications/>

Model Law Website: https://ncees.org/wp-content/uploads/Model_Law_2018.pdf

(3) You should also read through the Model rules. The model rules are published by NCEES and it is on the NCEES website. The link is shown below. The model rules complement the model laws. One key section in the model rules is the Rules of Professional Conduct.

Model Rules Website: https://ncees.org/wp-content/uploads/Model_Law_2018.pdf

Once you have read through these two writings, the following practice problems will help you to become familiar with the problems that may appear on the exam.

(4) This final task is optional, because the first three tasks should give you enough preparation for these 4-6 problems. The National Society of Professional Engineers or NSPE has information on a lot of legal cases and can provide insight on the topic of ethics and professional



practice. The following link has a list of Ethics Resources. You should read the NSPE Ethics Reference Guide.

NSPE Ethics Resources:

<https://www.nspe.org/resources/ethics/ethics-resources/other-ethics-resources>

NSPE Ethics Reference Guide:

<https://www.nspe.org/sites/default/files/resources/pdfs/Ethics/CodeofEthics/NSPECodeofEthicsforEngineers.pdf>

1.2 PUBLIC HEALTH, SAFETY & WELFARE

An engineer has the knowledge and duty to protect the public. The best resource for learning about an engineer's duty to public health, safety and welfare are shown on the National Society of Professional Engineer's website and specifically the codes of ethics. If an engineer is an expert in a certain topic and has facts about a situation that goes against a building code, plumbing code or any other engineering design code that puts the public's health, safety or welfare at risk, then the engineer has an ethical responsibility to report the situation to the appropriate parties.

<https://www.nspe.org/resources/ethics/ethics-resources/board-ethical-review-cases/public-health-and-safety-engineer-s>

An engineer must always design to the appropriate engineering standards applicable to the situation. If a client insists on designing below the minimum engineering standards, then the engineer must withdraw from the design and inform the proper authorities.

1.3 INTELLECTUAL PROPERTY

Intellectual property in engineering includes copyrights, patents, trade secrets and trademarks. For the purposes of the exam, you should understand the difference between each type of intellectual property and be able to come up with examples for each type of intellectual property. An excellent source to learn about intellectual property is on the United States Patent and Trademark Office website. <https://www.uspto.gov/ip-policy>

1.3.1 Copyrights

Copyrights are used to grant exclusive rights to artistic or intellectual work. Books, music, lyrics, literary works, dramatic works, movies, software and even architectural designs. Websites, literary writing, artwork and photographs can also be copyrighted. Recipes, names, titles, slogans, logos, ideas cannot be copyrighted. You can access copyright.gov for more information on what can and cannot be copyrighted. <https://www.copyright.gov/>



4 – Engineering Economics



Section 5.0 – Engineering Economics

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1.0 INTRODUCTION

Engineering Economics accounts for approximately 4 to 6 questions on the Mechanical FE exam. As an engineer, you will be tasked with determining the course of action for a design. Often times this will entail choosing one alternative instead of several other design alternatives. You need to be able to present engineering economic analysis to their clients in order to justify why a certain alternative is more financially sound than other alternatives. The following topics will present only the engineering economic concepts that you need for the FE exam and does not present a comprehensive look into the study of engineering economics. For the FE exam you are required to know the following concepts shown in the table below. Applicable equations for these topics can be found in the Engineering Economics section of the *NCEES FE Reference Handbook*.

Section 5.0 Engineering Economics (4 to 6 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
5A	Section 2.0	Time value of money (e.g., equivalence, present worth, equivalent annual worth, future worth, rate of return, annuities)

5B	Section 3.0	Cost types and breakdowns (e.g., fixed, variable, incremental, average, sunk)
5C	Section 4.0	Economic analyses (e.g., cost-benefit, break-even, minimum cost, overhead, life cycle)
	Section 5.0	Depreciation (not on FE Mechanical outline, but included just for your information)
	Section 6.0	Practice Exam Problems

2.0 TIME VALUE OF MONEY

The concept of time value of money is that money today is worth more than money in the future, because of the interest that could be earned by having the money today.

2.1 FUTURE AND PRESENT VALUE

Having money today is worth more than having that same amount of money in the future, due to the interest that could be earned. This is the time value of money concept. For example, if you were given the option to have \$1,000 today or to have \$1,000 ten years from now, most people will choose \$1,000 today, without understand why this option is worth more. The reason \$1,000 today is worth more is because of what could have done with that money; in the financial world, this means the amount of interest that could have been earned with that money. If you took \$1,000 today and invested it at 4% per year, you would have \$1,040 dollars at the end of the first year.

$$\$1,000 \times (1 + .04) = \$1,040$$

- If you kept the \$1,040 in the investment for another year, then you would have \$1,081.60.

$$\$1,040 \times (1 + .04) = \$1,081.60$$

- At the end of the 10 years the investment would have earned, \$1,480.24.

$$\$1,000 \times (1 + .04) \times (1.04) \times (1.04) \dots = \$1,000 \times (1.04)^{10} = \$1,480.24$$

- An important formula to remember is the Future Value (FV) is equal to the Present Value (PV) multiplied by (1+interest rate), raised to the number of years.

$$PV \times (1 + i)^{10} = FV$$

- As an example, what would be the present value of \$1,000, 10 years from now, if the interest rate is 4%?

$$PV \times (1 + .04)^{10} = \$1,000$$

$$PV = \$675.46$$

- Thus in the previous example, receiving \$1,000, 10 years from now, is only worth \$675.46 today.

It is important to understand present value because when analyzing alternatives, cash values will vary with time and the best way to make a uniform analysis is to *first convert all values to consistent terms, like present value.*

For example, if you were asked whether you would like \$1,000 today or \$1,500 in ten years (interest rate at 4%), then it would be a much more difficult question than the previous question. But with an understanding of present value, the "correct" answer would be to accept \$1,500 ten years from now, because the \$1000 today at 4% interest is only worth \$1,480 ten years from now. In this example, the \$1,000 today was converted to its future value 10 years from now. Once this value was converted, it was then compared to the \$1,500, which was presented as future value in 10 years. Notice how all values were converted to *future value* for comparison.

2.2 ANNUAL VALUE OR ANNUITIES

The previous section described the difference between present value and future value. It also showed how a lump sum given at certain times are worth different amounts in present terms. In engineering, there are often times when annual sums are given in lieu of one time lump sums. An example would be annual energy savings due to the implementation of a more efficient system. Thus, it is important for the engineer to be able to determine the present/future value of future **annual** gains or losses.

For example, let's assume that a solar hot water project, provides an annual savings of \$200. Using the equations from the previous section, each annual savings can be converted to either present or future value. Then these values can be summed up to determine the future and present value of annual savings of \$200 for four years at an interest rate of 4%.

Often times the engineer must convert money at various times, like annual, future or present. This section will take you through these important calculations. But first, another way of presenting future value is through the term salvage value.

Salvage value is the amount a piece of equipment will be worth at the end of its lifetime. Lifetime is typically given by a manufacturer as the average lifespan (years) of a piece of equipment. Looking at the figure below, initial cost is shown as a downward arrow at year 0. Annual gains are shown as the upward arrow and maintenance costs and other costs to run the piece of equipment are shown as downward arrows starting at year 1 and proceeding to the end of the lifetime. Finally, at the end of the lifetime there is an upward arrow indicating the salvage value.

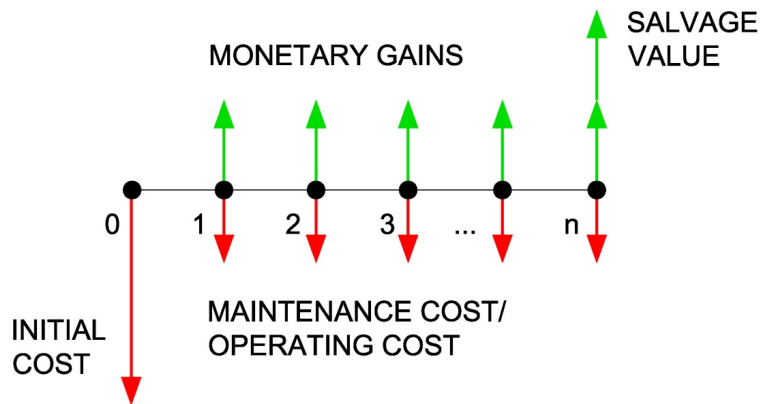


Figure 2: Adding salvage value to cash flow diagram.

As previously stated, the most important thing in engineering economic analysis is to *convert all monetary gains and costs to like terms*, whether it is present value, future value, annual value or rate of return. Each specific conversion will be discussed in the following sections.

Each of the following sections will use the same example, in order to illustrate the difference in converting between each of the different terms.

Example: A new chiller has an initial cost of \$50,000 and a yearly maintenance cost of \$1,000. At the end of its 15 year lifetime, the chiller will have a salvage value of \$5,000. It is estimated that by installing this new chiller, there will be an energy savings of \$5,000 per year. The interest rate is 4%.

5 – Electricity & Magnetism



Section 6.0 – Electricity and Magnetism

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1.0 INTRODUCTION

Electricity and Magnetism accounts for approximately 5 to 8 questions on the Mechanical FE exam. The mechanical discipline intersects with the electrical when mechanical energy is converted to electrical and vice versa. Hence, the mechanical engineer should have a basic understanding of electricity and magnetism. The most common application is the production of power via a generator and the reception of power through equipment via motors. Other applications include control circuits, variable frequency drives, and the effects of mechanical equipment on power quality. Remember that only the basics of the “Electricity and Computer Engineering” section of the *NCEES FE Reference Handbook* will be tested during the exam. The questions must also relate to the topics below.

Section 6.0 Electricity & Magnetism (5 to 8 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
6A	Section 2.0	Electrical Fundamentals (e.g., charge, current, voltage, resistance, power, energy, magnetic flux)
6B	Section 3.0	DC circuit analysis (e.g., Kirchhoff's laws, Ohm's law, series, parallel)
6C	Section 4.0	AC circuit analysis (e.g., resistors, capacitors, inductors)
6D	Section 5.0	Motors and generators
	Section 6.0	Practice Exam Problems

2.0 ELECTRICAL FUNDAMENTALS

2.1 CHARGE

An electric charge, Q , describes the number of electrons or protons there are. It can be positive (protons) or negative (electrons) and is measured in Coulombs (C). For example, one electron has -1.6×10^{-19} C of charge and one proton has $+1.6 \times 10^{-19}$ C. The movement of electrons is the foundation of how electricity works. It is unlikely that charge itself will be tested. It is more important to understand how charge is used to describe other concepts like current, power, voltage, and energy.

2.2 CURRENT

Current, I , is the movement of charge and is more specifically defined as the rate at which charge flows. It is represented in terms of Amps, where one amp is equal to the movement of one Coulomb of charge per second.

$$\text{Current, } I(\text{Amps}) = \frac{\text{Charge (C)}}{\text{Time (sec)}}$$

$$I = \frac{dq}{dt} \rightarrow \text{Charge, } Q = \int_{t_1}^{t_2} i(t) dt$$

For steady flow, current can be calculated as:

$$I (\text{Amps}) = \frac{\Delta Q}{\Delta t}$$

One characteristic to distinguish is that current flows in the opposite direction of electrons. Current flows from positive to negative, see the green arrow in the figure below, start at the positive end of the battery, loop around the circuit and end at the negative end. Electrons on the other hand are attracted to positive charge, so it will flow from negative to positive, as shown in red below.

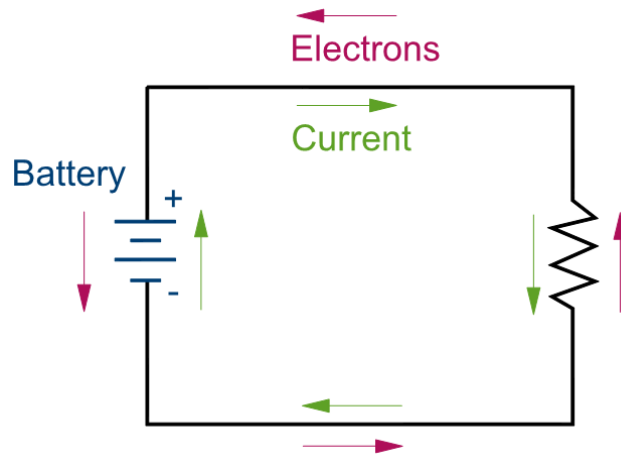


Figure 1: Current flows in a circuit from the positive end of the battery to the negative, as shown in green, while electrons flow from negative to positive.

Direct current (DC) is the supply of current in one direction. As mentioned previously, current flows from the positive voltage terminal to the negative terminal in a circuit. Current is deemed positive when it flows in this direction. Current is considered negative when it flows from a negative terminal to a positive terminal. DC current is a constant source and does not switch between negative and positive. The simplest example of a DC source is a battery.

Alternating current (AC) is able to supply current in both directions, positive to negative and negative to positive. This is shown in the graph below, where the current can be positive (above the 0-axis) or negative (below the 0-axis). Alternating current is what is supplied by the electric company to buildings. Alternating current is further discussed in the Alternating Circuits topic.

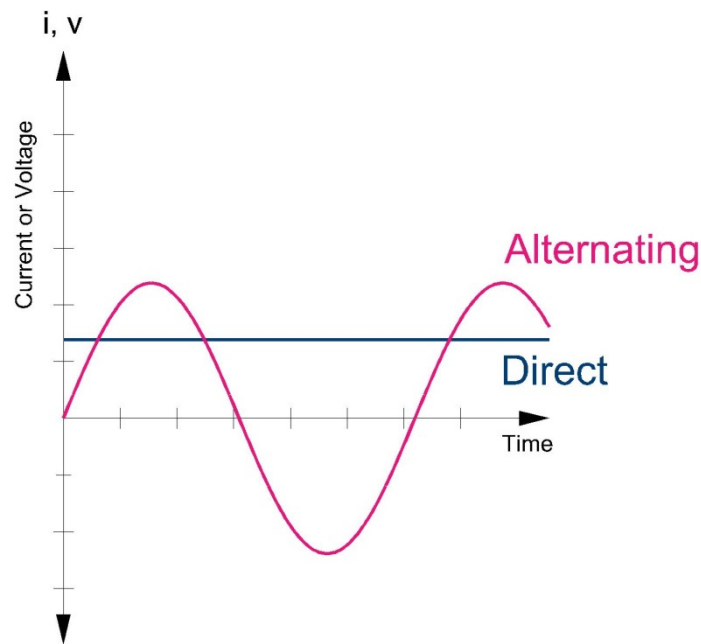


Figure 2: In an AC circuit, current can alternate its flow from positive to negative. In a DC circuit, current is constant.

2.3 VOLTAGE

Voltage, V is the potential energy in electricity; it is the amount of energy held in one charge and is given in units of volts.

$$\text{Voltage (V)} = \frac{\text{Potential Energy (J)}}{\text{Charge (C)}}$$

Voltage is measured between two points because potential energy is the difference in energy. This potential energy in a circuit is what drives the flow of electrons, and therefore the current, from one point to the next. A battery is typically a source voltage, supplying energy into a circuit. A voltage can also be measured across a load or a resistor, this is known as voltage drop since the energy is being absorbed by the load.

2.4 RESISTORS

To complete a circuit, wires are connected from a voltage source to a load, which is represented by the resistance. Resistance is measured in units of ohms (Ω). It is the opposition to the flow of current. One ohm is described as the level of resistance that will allow 1 ampere to flow when 1 volt is applied to a circuit. The following figure depicts a basic circuit with current flowing from a voltage source, then through a resistor.

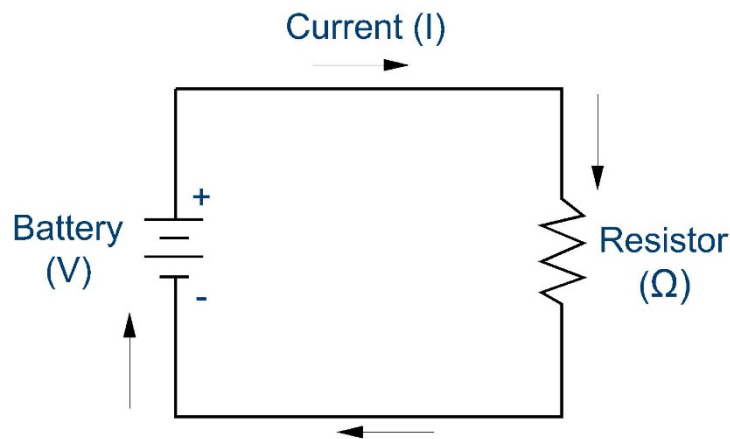


Figure 3: A basic DC circuit, current flows from positive to negative.

Ohm's law describes the relationship between voltage, resistance and current. The voltage in a circuit is equal to the product of the current and the resistance in the circuit.

$$\text{Ohm's Law} \rightarrow V = IR$$

$$V = \text{voltage (volts)}; I = \text{current (amperes)}; R = \text{ohms } (\Omega)$$

As voltage increases, in a constant resistance circuit, current will rise. As current increases, in a constant resistance circuit, voltage will rise.

2.5 POWER

The next concept that you must understand is electrical power. Power is the rate at which energy flows and is given in units of Watts (Joules per second). The power equation in basic circuits is given below and states that real power is equal to the product of the voltage and the current. Real power is given in terms of watts.

$$\text{Real Power} \rightarrow P = IV$$

$$P = \text{real power (watts)}$$

If you combine Ohm's law and the real power equation, then you will find two equations that show power as a function of resistance. These equations, along with Ohm's Law can be found in the *NCEES FE Reference Handbook*.

$$P = I^2R; P = \frac{V^2}{R}$$

6 - Statics



Section 7.0 – Statics

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1.0 INTRODUCTION

Statics accounts for approximately 9 to 14 questions on the Mechanical FE exam. These questions can cover statics, but not dynamics. The statics topic on the NCEES exam is similar to a common statics college engineering class. Statics is the study of components at equilibrium, which means the components are at rest or at zero acceleration. This topic includes vectors, free body diagrams, moments, reaction forces, first moment of area, static friction and second moment of area. These concepts and skills are used to solve problems on pulleys, cables, springs, beams, trusses, frames, etc.

The *NCEES FE Reference Handbook* Statics section has some basic equations for the topics below, but it does not explain the skills and concepts necessary to use these equations. You should learn the skills and concepts presented in this section and go through the handbook to confirm that you know how to use the basic equations. You may also need to know some of the Mathematics equations like law of cosines and other trigonometry equations presented in the Mathematics section. The handbook also presents screw threads but this is covered more in *Section 14.0 Mechanical Design and Analysis* in this book.

Section 6.0 Statics (9 to 14 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
6A	Section 2.0	Resultants of Force Systems
6B	Section 3.0	Concurrent Force Systems
6C	Section 4.0	Equilibrium of Rigid Bodies
6D	Section 5.0	Frames and Trusses
6E	Section 6.0 & 7.0	Centroids & Moments of Inertia
6F	Section 8.0	Static Friction
	Section 9.0	Practice Exam Problems

2.0 RESULTANTS OF FORCE SYSTEMS

As previously stated, statics is the study of mechanical components at equilibrium, which means the components have zero acceleration or are at rest. The material presented on statics focuses on the key equations and skills necessary to complete the possible problems within this topic on the FE exam.

2.1 VECTORS

Vectors are used to visualize forces and moments in this section and in *Section 8.0 Mechanics of Materials* and they are used to visualize movement in *Section 7.0 Dynamics, Kinematics and*

Vibration. This section will focus on vectors being used to describe the magnitude and direction of force or moment. On the FE exam you must be able to translate words or diagrams into force/moment vectors and you must be able to add/subtract vectors and multiply/divide vectors by scalars.

Vectors can either be represented in a rectangular form or a polar form. The rectangular form consists of an x-component and a y-component. These values are used to represent the magnitude in the x and y directions. In real applications, there will also be a z-component, but for the purposes of the exam you most likely will only need the x and y components. The polar form is shown as a magnitude and an angle. The magnitude describes the length of the vector while the angle determines the direction.

Vector forms are discussed in the Mathematics section of this book, but is repeated here for completeness.

2.1.1 Rectangular Form

The rectangular form is shown as, x plus the y component. The y component is shown as j and the x component is shown as i.

Rectangular form: $xi + yj$

$x = x - \text{component}; y = y - \text{component}$

Example A $\rightarrow 5i + 2j$ or Example B $\rightarrow -1i - 5j$

The rectangular form is used when adding and subtracting vectors and follows the same rules as normal addition and subtraction, where only like terms can be added and subtracted. For example, “Example A” plus “Example B”, is solved with the following process.

Example A + Example B = $(5i + 2j) + (-1i - 5j)$

$(5i - 1i) + (2j - 5j) \rightarrow 4i - 3j$

The rectangular form can also be understood via a graphical format, where the x-axis represents the real component and the y-axis represents the imaginary component.

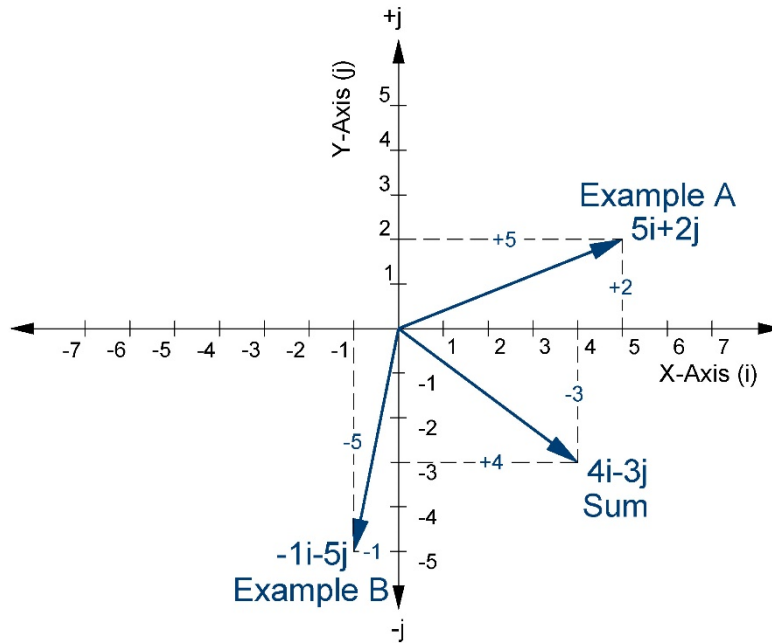


Figure 1: Example “A” vector, example “B” vector and the sum of the two vectors is shown in the above graph.

2.1.2 Polar Form

The polar form is best understood in its graphical format. The format consists of a phasor magnitude at a phasor angle relative to the x-axis.

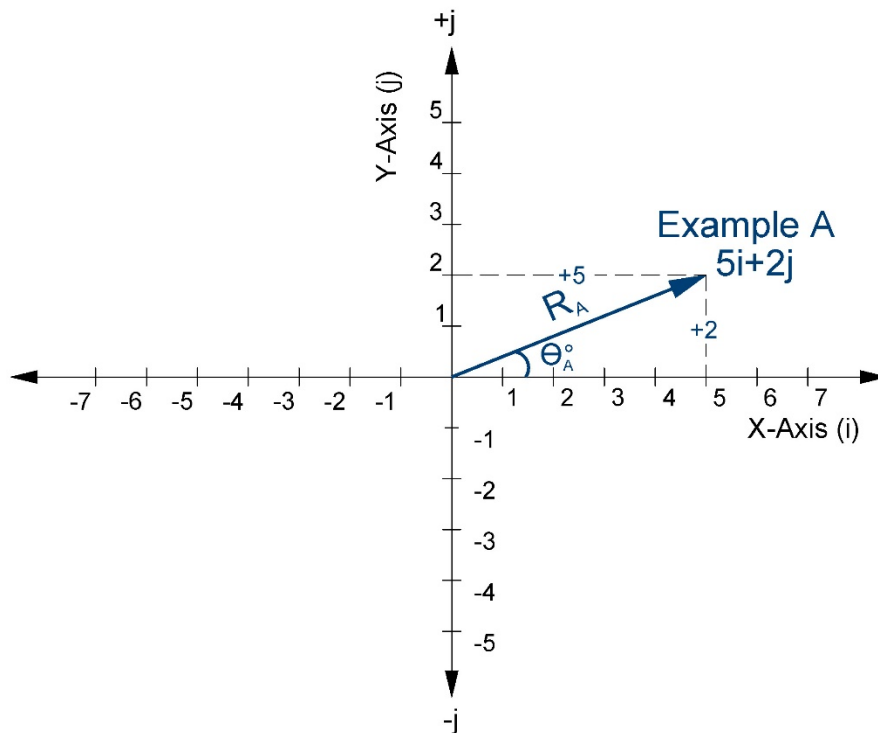


Figure 2: A phasor shown in both polar and rectangular form.

Polar form: $R\angle\theta^\circ$; R = phasor magnitude; θ = phasor angle relative to x – axis

$$\text{Example A: } R_A = \sqrt{5^2 + 2^2} = 5.4; \theta_A = \tan^{-1}\left(\frac{2}{5}\right) = 21.8^\circ$$

In the above example, the polar form is converted from the rectangular form by using the Pythagorean Theorem to find the radius (i.e. the magnitude) and the inverse tangent to find the angle. The polar form is not typically used for adding or subtracting, but it is used for multiplication and division. When multiplying or dividing two polar forms, you must multiple/divide the radiuses and add or subtract the angles. If the polar forms are being multiplied, then you must add the angles and if you are dividing one polar form from another then you subtract the divisor from the dividend.

$$5.5\angle 45^\circ \div 1.1\angle 85^\circ \rightarrow \left(\frac{5.5}{1.1}\right)\angle(45 - 85^\circ) \rightarrow 5\angle -40^\circ$$

2.1.3 Converting Polar and Rectangular Forms - Calculator

During the exam, you will need to convert from polar form to rectangular form and vice versa. You will need to convert between the two forms in order to carry out multiplication/division or addition/subtraction. You should be able to quickly convert between the forms with your calculator. This will help to save you time for more difficult tasks during the exam.

7 – Dynamics, Kinematics & Vibrations



Section 8.0 – Dynamics, Kinematics and Vibrations

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1.0 INTRODUCTION

Dynamics, Kinematics and Vibrations accounts for approximately 9 to 14 questions on the Mechanical FE exam. This section is first broken down into the two topics, (1) particles and (2) rigid bodies. Particles are a single mass subject and rigid bodies consist of a collection of particles into a solid body that does not deform. The kinematics, work, energy, impulse and momentum and kinetics equations first focus on a single mass subject. You may be familiar with problems that had a block moving down a hill or a ball being thrown. These types of problems covered particles. Problems on rigid bodies are those that contain mechanical components like cams where one part of the body moves relative to another part of the body.

Kinematics covers the movement, speed and acceleration of particles and rigid bodies. This includes radial movement and movement due to gravity. Kinetics builds upon kinematics by including force and energy, which also transitions into the work-energy topic. Lastly, this section covers friction and impulse-momentum.

Section 8.0 Dynamics, Kinematics & Vibrations (9 to 14 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
8A	Section 2.0	Kinematics of Particles
8B	Section 3.0	Kinetic Friction
8C	Section 4.0	Newton's Second Law for Particles
8D	Section 5.0	Work-Energy of Particles
8E	Section 6.0	Impulse-Momentum of Particles
8F	Section 7.0	Kinematics of Rigid Bodies
8G	Section 8.0	Kinematics of Mechanisms
8H	Section 9.0	Newton's Second Law for Rigid Bodies
8I	Section 10.0	Work-Energy of Rigid Bodies
8J	Section 11.0	Impulse-Momentum of Rigid Bodies
8K	Section 12.0	Free and Forced Vibrations
	Section 13.0	Practice Exam Problems

2.0 KINEMATICS OF PARTICLES

Kinematics type problems will be centered on finding one of these three variables, (1) the distance traveled by an object, (2) the velocity of an object or (3) the acceleration of an object at any given time or location. If a problem asks for one of these three variables and forces are not involved, then most likely the solution will be found using the equations presented in the kinematics topic.

Within the topic of kinematics you may encounter problems with either linear motion or angular motion. Linear motion is the movement of an object within the x-y-z plane in either a straight line or a curve. Curve type movement is typical of projectiles and straight line movement is typical of



vehicles, sliding blocks, pistons and springs. Angular motion is the circular movement of an object about an axis, within the x-y-z plane. This type of movement is typical of gears, pumps, fans and any other equipment that rotates about an axis.

2.1 LINEAR MOTION

Linear motion can be characterized by its displacement, velocity and acceleration equations.

2.1.1 Linear Displacement

The first equation in linear motion is displacement, represented by the variable, x , as a function of time, t , also expressed as the function, $x(t)$. The initial position is given as x_i and the final position is represented as x_f . If the velocity of a particle is constant, then the acceleration is zero and the final position can be expressed as the following equation.

$$\text{Constant velocity} \rightarrow x_f = x_i + vt; a = 0;$$

On the FE exam, you can use the following equations to help you solve any kinematics problems for the final position (x_f) when there is constant acceleration.

Solving for Distance with Uniform Acceleration ($a=\text{constant acceleration}; V_i=\text{initial velocity}$)			
	When $x_i = 0 \Rightarrow$		
$x_f = x_i + V_i t + \frac{1}{2} a t^2$	$x_f = \frac{(V_f + V_i)t}{2}$	$x_f = \frac{(V_f^2 - V_i^2)}{2a}$	$x_f = V_f t - \frac{1}{2} a t^2$

2.1.2 Linear Velocity

The instantaneous velocity at a time, t , is the derivative of the position.

$$v(t) = \frac{dx}{dt}; \text{ Units} \rightarrow \frac{m}{s} \text{ or } \frac{ft}{s}$$

The average velocity of an object can be found by dividing the change in position over a specific time interval.

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

On the FE exam, you may use the following equations to help you solve any kinematics problems for the specific scenario of uniform acceleration.

Solving for Velocity with Uniform Acceleration ($a=\text{constant acceleration}; V_i=\text{initial velocity}; x_i=0$)			
$V_f = V_i + at$	$V_{avg} = \frac{x_f - x_i}{t}$	$V_f^2 = V_i^2 + 2a(x_f - x_i)$	$V_f = \sqrt{V_i^2 + 2ax_f}$



2.1.3 Linear Acceleration

The instantaneous acceleration at a time, t , is the derivative of the velocity or the second derivative of the position.

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}; \text{ Units} \rightarrow \left[\frac{m}{s^2} \right] \text{ or } \left[\frac{ft}{s^2} \right]$$

The average acceleration of an object can be found by dividing the change in velocity over a specific time interval.

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t}$$

If the object is increasing in speed (accelerating), then acceleration will be positive. If the object is decreasing in speed (decelerating), then acceleration will be negative. If the object is moving at constant velocity then acceleration will be equal to 0.

Solving for Acceleration with Uniform Acceleration ($a=\text{constant acceleration}$; $V_i=\text{initial velocity}$; $x_i=0$)			
$a = \frac{(V_f^2 - V_i^2)}{2x_f}$	$a = \frac{v_f - v_i}{t}$	$a = \frac{2(x_f - V_i t)}{t^2}$	$a = \frac{2(V_f t - x_f)}{t^2}$

2.1.4 Projectiles

When completing these types of problems, realize that motion can occur in the x, y, and z-axis and equations must be produced for each axis. For example, projectiles can move in the x and y-axis at the same time, but acceleration due to gravity only occurs in the y-axis. The previously discussed equations can be re-written for the x and y-axis, based on the assumption that there is no x-acceleration and that the only y-acceleration is downward (negative) due to gravity.

x-axis ($a = 0$)	y-axis ($a = \text{gravity} = -g$)
$V_{f,x} = V_{i,x} + at$	$V_{f,y} = V_{i,y} - gt$
$x_{f,x} = x_{f,x} + V_{f,x}t - \frac{1}{2}at^2$	$x_{f,y} = x_{f,y} + V_{f,y}t - \frac{1}{2}gt^2$
$V_{f,x} = \sqrt{V_{i,x}^2 + 2ax_f}$	$V_{f,y} = \sqrt{V_{i,y}^2 - 2gx_f}$

2.2 ANGULAR MOTION

Similar to linear motion, angular motion can be characterized by its equations governing displacement, velocity and acceleration.



8 – Mechanics of Materials



Section 9.0 – Mechanics of Materials

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1.0 INTRODUCTION

Mechanics of Materials accounts for approximately 9 to 14 questions on the Mechanical FE exam. Mechanics of Materials questions basically will cover calculating the stress or strain due to different loadings, like axial loads, bending loads, torsional loads and shear loads. In addition, you will have to calculate the displacement, shear force and moments for beams and the buckling forces and stresses in columns.

The equations shown in the *NCEES FE Reference Handbook* on Mechanics of Materials is comprehensive and all of these equations are fairly simple and easy to use. This means that the equations do not have any complex math. After going through this section, you should be familiar with the different types of loadings and all the equations within the handbook and should be able to quickly recognize when to use each equation for each problem.

Section 9.0 Mechanics of Materials (9 to 14 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
9A	Section 2.0	Shear and Moment Diagrams
9B	Section 3.0 & 4.0	Stress Transformations & Mohr's Circle
9C	Section 5.0	Stress and Strain caused by Axial Loads
9D	Section 6.0	Stress and Strain caused by Bending loads
9E	Section 7.0	Stress and Strain caused by Torsion
9F	Section 8.0	Stress and Strain caused by Shear
9G	Section 9.0	Stress and Strain caused by Temperature Change
9H	Section 10.0	Combined loading
9I	Section 11.0	Deformations
9J	Section 12.0	Column Buckling
9K	Section 13.0	Statically Indeterminate Systems
	Section 14.0	Practice Exam Problems

2.0 SHEAR AND MOMENT DIAGRAMS

Shear and moment diagrams are used to graphically show how forces, displacement and moments change depending on the location within a component. The components that you will most likely need to know are beams, since the shear and moment diagrams included in the *NCEES FE Reference Handbook* are only for beams.

However, before you use the diagrams you should understand the concepts behind the beam diagrams and the resulting equations. Once you have a good grasp on the concepts and how to use the equations, then you should be able to solve these types of problems.

There are three main types of beam diagrams, (1) Free Body Diagram, (2) Shear Diagram and (3) Moment Diagram. The first step (1) is to determine the forces acting upon the beam in order to construct the beam diagram. Beams can be loaded with a load at a point or a distributed load along the entire length of the beam. The loads are primarily downward and in order to create equilibrium there will be reaction forces upward at the supports. Equilibrium equations for both forces and moments about each of the supports are used to find the reaction forces. The forces must equal zero, since the beam is restricted from movement. Also the sum of the moments at each of the supports must be zero, since the beam is restricted from twisting. Once you have found all loads and reaction forces, then you can construct the free body diagram.

The next step (2) is to construct the shear diagram. The shear diagram describes the internal forces within the beam at any point of the beam. This diagram is created by splitting up each segment separated by a point force or reaction force. Then you must cover all external forces to the right of each point and sum up all external forces and this sum is the shear force at that point.

The final step (3) is to construct the moment diagram. The moment diagram is comprised of the moment at any location along the length of the beam. The moment diagram is constructed in a similar fashion as the shear diagram.

In practice, shear diagrams and moment diagrams are not often re-constructed, since the diagrams have already been documented. Please see the *NCEES FE Reference Handbook* for completed beam diagrams with equations. Beam diagrams will most likely be used on the FE exam in order to find the maximum stress in the beam or the maximum deflection in the beam. This can be done by using the given beam diagrams and using the corresponding equations. However, there may be conceptual type problems which will require you to have an understanding of how these diagrams are constructed. The next few pages will show a few diagrams and walk you through some points on the diagram.

In practice, beam supports can be classified based on how much the beam is restricted by the support.

- Loose support (simply supported): A loose support, which would be the same as the simply support diagram, is used to support the weight vertically. The beam can be moved in all other directions. Including along the length of the beam (longitudinal) and perpendicular to the direction of the pipe (transverse). The beam is only restricted from moving vertically (up and down).
- Loose support with longitudinal guide: This type of support includes the loose support but also has a restriction on the transverse. The beam will not be allowed to move perpendicular to the direction of the beam. This would include putting a clamp around a beam to stop the side to side movement.

- Loose support with transverse guide: This type of support includes the loose support but also has a restriction on the longitudinal. This would stop the beam from expanding and contracting in the longitudinal direction.
- Loose support with anchor: This type of support restricts movement in both the longitudinal and transverse direction.

2.2 SIMPLE BEAM WITH A UNIFORMLY DISTRIBUTED LOAD

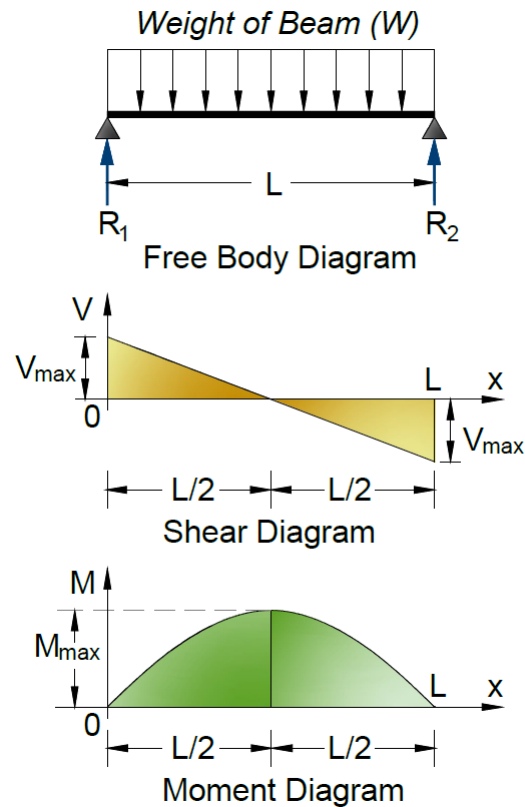


Figure 1: Simple beam with uniformed load diagrams

This diagram is typically used when a beam is supported at both ends for only the weight of the beam. The downward force is equal to the weight of the beam. The weight is evenly distributed evenly the entire length of the beam, which is why the vectors are of equal size.

$$\text{Total Weight of Beam} = W \text{ (lbf)} = w \text{ (lbf/ft)} * L \text{ (ft)}$$

$$\text{Total Weight of Beam} = W \text{ (N)} = w \text{ (N/m)} * L \text{ (m)}$$

There will be upward reaction forces at the support that will counteract the weight of the beam. The force at each support will be equal to one-half the entire weight.

9 – Material Properties & Processing



Section 9.0 – Material Properties and Processing

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1.0 INTRODUCTION

Material Properties and Processing accounts for approximately 7 to 11 questions on the Mechanical FE exam. This section works in conjunction with *Section 8 Mechanics of Materials*. The Material Properties part covers the mechanical properties covered in Section 8 and other material properties like chemical, electrical and physical types. This section then covers the material processing, material types, phase diagrams and other material issues like corrosion, fatigue and cracks.

There are a few pages on Material Properties & Processing in the *NCEES FE Reference Handbook* that you should be familiar with in order to pass the FE exam. However, in order to use those pages you need to understand the concepts and skills presented in this section.

Section 9.0 Material Properties & Processing (7 to 11 Problems)		
NCEES Outline Value	Engineering Pro Guides	
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9B	Section 3.0	Stress-Strain Diagrams
9C	Section 4.0	Ferrous Metals
9D	Section 5.0	Nonferrous Metals
9E	Section 6.0	Engineered Materials (e.g., composites, polymers)
9F	Section 7.0	Manufacturing Processes
9G	Section 8.0 & 9.0	Phase Diagrams, Phase Transformation and Heat Treating
9H	Section 10.0	Materials Selection
9I	Section 11.0	Corrosion Mechanisms and Control
9J	Section 12.0	Failure Mechanisms (e.g., thermal failure, fatigue, fracture, creep)
	Section 13.0	Practice Exam Problems



2.0 PROPERTIES (CHEMICAL, ELECTRICAL, MECHANICAL, PHYSICAL, THERMAL)

2.1 CHEMICAL

A chemical property is the property of a material that undergoes change in its chemical structure during a chemical reaction. An important chemical property is corrosion, which will be discussed later in this section.

2.2 ELECTRICAL

The important electrical properties of materials are conductivity and capacitance. The conductivity describes the material's ability to conduct electricity and capacitance describes the material's ability to hold a charge. Conductivity is often expressed as its inverse, which is called resistivity. This is the material's ability to resist an electrical current.

$$\text{Conductivity}(C) = \frac{1}{\text{Resistivity}(\rho)}$$

Resistivity is used to calculate the resistance of an object like a copper wire. Resistance is given in units of ohms and is a function of the material's resistivity, the cross sectional area of the material and the length of the material.

$$R(\Omega) = \frac{\rho L}{A}$$

$$A = \text{area (m}^2\text{)}; l = \text{length (m)}$$

$$\rho = \text{resistivity } (\Omega - \text{m}); C = \text{conductivity } \left(\frac{1}{\Omega - \text{m}}\right)$$

2.3 MECHANICAL

The mechanical properties of materials are found by a series of tests. These tests include the tension or compression test, which determines the Stress-Strain diagram. The Rockwell or Brinell tests determine the hardness of the material. There are also many more tests that determine the other properties like creep, thermal expansion, thermal conductive, specific heat capacity and visco-elasticity. Each of the mechanical material properties will be discussed in the next paragraphs.

2.3.1 Strength

Strength is covered in *Topic 3.0 Stress-Strain Diagrams*.



2.3.2 Hardness

Hardness is the property that describes a material's ability to withstand abrasion, scratching and indentation. Hardness is measured in terms of MPa or is a dimensionless value. The hardness measurements are useful when comparing one material to another.

Hardness is measured by any one of the following tests, (1) Rockwell, (2) Brinell, (3) Meyer, (4) Vickers, (5) Knoop or (6) Scleroscope. Each of these tests use a different object of varying shape and material to impact the subject material. The effect of this impact due to a known force is then measured and a hardness value is assigned. The two main tests that you should know for the FE exam are Rockwell and Brinell. You should also know that hardness values are not absolute, these tested values are relative to each other. Hardness values are used to compare the hardness between different materials.

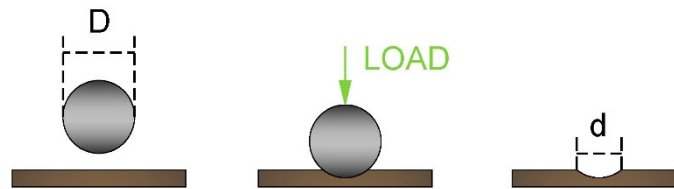


Figure 1: Hardness tests involve a known load impacting a material's surface and the measurement of the load and the result of the impact upon the material's surface.

2.3.1 Rockwell

The Rockwell hardness test is the most popular test. In this test, an indenter is first used as a pre-load to remove any effects of the surface finish and then the full load is used to indent the material. The depth of the indentation determines the Rockwell hardness value. This test has different scales based on the size of the indenter used to impact the test material. The scales range from A through G. Each scale is used for different types of materials. A summary of the scales are shown below.

Rockwell Hardness Scale	Full Load Value (kg)	Indenter Size	Typical Use
A	60	120° cone, 0.2 mm radius diamond	Extremely hard materials
B	100	1/16"-D Steel Sphere	Soft steel
C	150	120° cone, 0.2 mm radius diamond	Hard steel, iron
D	100	120° cone, 0.2 mm radius diamond	Medium hardness
E	100	1/8"-D Steel Sphere	Medium Copper, aluminum
F	60	1/16"-D Steel Sphere	Soft Copper, aluminum
G	150	1/16"-D Steel Sphere	Softest materials

The hardness number is then determined by measuring the thickness of the indent made into the material and inserting this value into one of the following equations.

$$\text{Rockwell A, C and D Hardness \#} = 100 - 500t$$



2.3.2 Brinell

The Brinell hardness test is another popular hardness test method. This test uses a 10-mm diameter steel ball and a 3,000 kg force. As the materials get softer, a smaller force is used and for harder materials, a tungsten carbide ball is used instead of steel.

$$\text{Brinell Hardness (BHN)} = \frac{2P}{\pi D(D - \sqrt{D^2 - d^2})}$$

$$\text{Brinell Hardness} \left[\frac{\text{kgf}}{\text{mm}^2} \right]; P = \text{kgf};$$

$$D = \text{diameter of indenter [mm]}; d = \text{diameter of indentation [mm]}$$

There is a rough relationship between the Brinell Hardness Number and the tensile stress of a material, which is shown with the equation below.

$$\text{Tensile Stress (psi)} = 500 * (\text{BHN})$$

$$\text{Tensile Stress (MPa)} = 3.5 * (\text{BHN})$$

2.4 PHYSICAL

During the exam you will need to be able to find and use material properties to complete many problems. You should be very familiar with your resources and where to find these material properties. As you go through these descriptions of the important material properties, look through the *NCEES FE Reference Handbook* so you can become familiar with the available properties and the units.

2.4.1 Density

The density of a substance is its mass per unit volume, basically how heavy is something in one cubic foot or one cubic meter.

$$\text{density, } \rho = \frac{\text{mass}}{\text{Volume}} \left[\frac{\text{lbm}}{\text{ft}^3}, \frac{\text{lbm}}{\text{in}^3}, \frac{\text{kg}}{\text{m}^3} \text{ or } \frac{\text{g}}{\text{cm}^3} \right]$$

Common Conversion Factors:

$$1 \frac{\text{lbm}}{\text{ft}^3} = 0.000578 \frac{\text{lbm}}{\text{in}^3}$$

$$1 \frac{\text{kg}}{\text{m}^3} = 0.001 \frac{\text{g}}{\text{cm}^3} = 0.0624 \frac{\text{lb}}{\text{ft}^3}$$



10 – Fluid Mechanics



Section 11.0 – Fluid Mechanics

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1.0 INTRODUCTION

Fluid mechanics accounts for 10 to 15 problems on the Mechanical FE Exam. The topics range from college fluid mechanics topics like fluid properties, fluid statics, energy, impulse, momentum, internal flow, external flow and compressible flow to the topics that are more often used in practice like incompressible flow, power, efficiency, performance curves and scaling laws for fans, pumps and compressors. As you go through this section, you should also check the fluids topics within the *NCEES FE Reference Handbook*.

Section 11.0 Fluid Mechanics (10 to 15 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
11A	Section 2.0	Fluid Properties
11B	Section 3.0	Fluid Statics
11C	Section 4.0	Energy, Impulse and Momentum
11D	Section 5.0	Internal Flow
11E	Section 6.0	External Flow
N/A	Section 7.0	Incompressible Flow
11F	Section 8.0	Compressible Flow
11G	Section 9.0	Power and Efficiency
11H	Section 10.0	Performance Curves
11I	Section 11.0	Scaling Laws for Fans, Pumps and Compressors
	Section 12.0	Practice Exam Problems

2.0 FLUID PROPERTIES

During the exam you will need to be able to find and use fluid properties to complete many problems. You should be very familiar with the *NCEES FE Reference Handbook* and where to find these fluid properties. As you go through these descriptions of the important fluid properties, look through the handbook and you will see that the only fluids mentioned in the handbook with all of these properties are air and water. This should give you an indication that most of the questions on fluids will revolve around air and water and if another fluid is given in a question, then all the properties for that fluid must be provided in the question, except for heat capacity for select fluids like air, water, ethane, methane, mercury, etc. Only select fluids have their densities shown in the handbook.

2.1 DENSITY

The density of a substance is its mass per unit volume. For example, density is typically shown as pound-mass per cubic foot or kilograms per cubic meter.

$$\text{Density} = \frac{\text{lbm}}{\text{ft}^3} [\text{IP}] \text{ or } \frac{\text{kg}}{\text{m}^3} [\text{metric}]$$

The density of a fluid is measured as a weight per unit volume. Specific volume is the inverse of density and is measured as a volume per unit mass.

$$\text{Specific Volume} = \frac{\text{ft}^3}{\text{lbm}} [\text{IP}] \text{ or } \frac{\text{m}^3}{\text{kg}} [\text{metric}]$$

2.1.1 IP Conversions

When performing calculations in English units, it is important to distinguish between pound mass (lbm) and pound force (lbf). The mass of an object is measured in pound-mass, similar to the English units of kilograms (kg). Pound-force, on the other hand, is a measurement of weight. It is a unit of force and is used to describe the mass of an object subject to gravity. Pound-force is comparable to the metric force of Newtons.

$$\text{Weight [lbf]} = \text{mass} * \text{gravity} = 1\text{lbm} * 32.174 \frac{\text{ft}}{\text{s}^2}$$

$$1\text{lbf} = 32.174 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2}$$

To perform calculations between pound-mass and pound force, the conversion factor, g_c is used. Since g_c is merely a unit conversion factor, it can be multiplied or divided anywhere in an equation.

$$g_c [\text{conversion factor}] = \frac{ma}{F}$$

$$g_c = 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}$$

Finally, mass can be represented in terms of slugs, which simplifies the force equations, essentially internalizing the g_c conversion factor.

$$1 \text{ slug} = 32.2 \text{ lbm}$$

$$1\text{lbf} = 1\text{lbm} * \frac{1\text{slug}}{32.2\text{lbm}} * 32.2 \frac{\text{ft}}{\text{s}^2} = 1\text{slug} * \frac{\text{ft}}{\text{s}^2}$$

$$1 \text{ slug} = \frac{1\text{lbf} \cdot \text{s}^2}{\text{ft}}$$

Example: Density relationships in terms of lbm, slug and lbf.

$$\text{Density, } \rho = 1 \frac{\text{lbm}}{\text{ft}^3} = \frac{\text{lbm}}{\text{ft}^3} * \frac{1}{g_c} = \frac{\text{lbm}}{\text{ft}^3} * \frac{\text{lbf} \cdot \text{s}^2}{32.2\text{lbm} \cdot \text{ft}} = \frac{1}{32.2} \frac{\text{lbf} \cdot \text{s}^2}{\text{ft}^4}$$

$$32.2 \frac{lbm}{ft^3} = 1 \frac{lb_f \cdot s^2}{ft^4} = 1 \frac{slug}{ft^3}$$

Realize that the densities in the *NCEES FE Reference Handbook* properties table are given in terms of $\frac{lb_f \cdot s^2}{ft^4}$ or $\frac{slug}{ft^3}$.

2.2 VISCOSITY

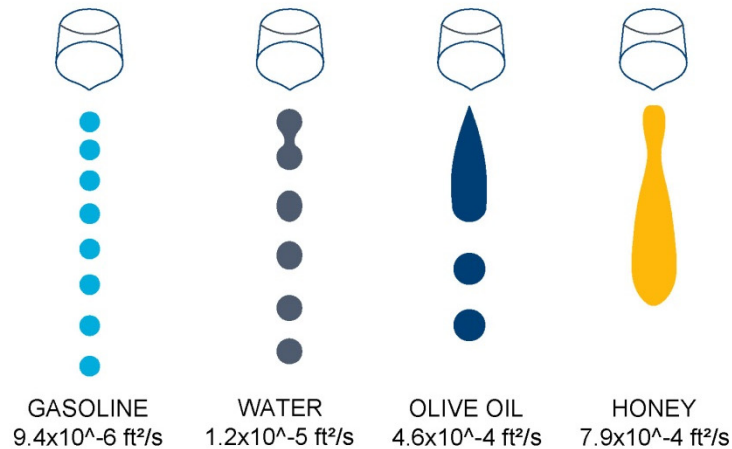


Figure 1: Varying liquids and their kinematic viscosities

The viscosity of a fluid describes the fluid's resistance to flow. Viscosity is measured in *cP* or centipoises and is represented by the variable, μ or μ . Viscosity is measured with a device called a viscometer. There are many different types of viscometers, but each typically has the fluid moving past/through an object or it has the object moving through the fluid. The time of travel will vary based on the viscosity of the fluid. For example, water has a viscosity of ~ 1.00 cP (centipoises) at 68° F, while syrup has a viscosity of ~ 1400 cP and air has a viscosity of ~ 0.01827 cP.

$$\mu = \text{viscosity, [cP]}$$

$$\text{Alternate units of } \mu: \left[\frac{lbm}{ft \cdot s} \right] \text{ or } \left[\frac{lb_f \cdot s}{ft^2} \right] \text{ or } \left[\frac{g}{cm \cdot s} \right] \text{ or } \left[\frac{N \cdot s}{m^2} \right]$$

The units described above are related to cP by a factor of 100. 100 cP is equal to 1 $[g/(cm \cdot s)]$. The imperial units are $[lbm/(ft \cdot s)]$ and are related to cP by the following conversion.

$$1 \text{ cP} = 6.72 \times 10^{-4} \left[\frac{lbm}{ft \cdot s} \right]$$

11 - Thermodynamics



Section 11.0 –Thermodynamics

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1.0 INTRODUCTION

Thermodynamics accounts for approximately 10 to 15 questions on the FE Mechanical exam. Thermodynamics is the largest topic on the FE Mechanical Exam. It contains so much information that most Mechanical Engineering college courses do not cover all of this information within their entire Mechanical Engineering degree course curriculum. 11G, 11I, 11J are most often left out of Thermodynamic college courses. In addition, many college courses cover a lot of theory and do not focus on the key concepts and skills that are necessary for practical application of the theory. The following list is the outline of the Thermodynamics topics that can appear on the exam. This section will go into detail on the key concepts and skills that are necessary under each topic. Following, the key concepts and skills there will be problems that you can use to test your understanding of the concepts and skills.

Section 11.0 Thermodynamics (10 to 15 Problems)		
NCEES Outline Value	Engineering Pro Guides	
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11A	Topic 2.0	Properties of Ideal Gases and Pure Substances
11B	Topic 3.0	Energy Transfers
11C	Topic 4.0	Laws of Thermodynamics
11D	Topic 5.0	Processes
11E	Topic 6.0	Performance of Components
11F	Topic 7.0	Power Cycles
11G	Topic 8.0	Refrigeration and Heat Pump Cycles
11H	Topic 9.0	Non-reacting Mixtures of Gases
11I	Topic 10.0	Psychrometrics
11J	Topic 11.0	Heating, Ventilating and Air-Conditioning (HVAC) Processes
11K	Topic 12.0	Combustion and Combustion Products
	Topic 13.0	Practice Exam Problems

2.0 PROPERTIES OF IDEAL GASES & PURE SUBSTANCES

This topic, Properties of Ideal Gases & Pure Substances, describes the typical properties for both gases and liquids. On the exam, you should be able to find thermodynamic properties very easily through the use of your *NCEES FE Reference Handbook* for given gases and liquids. The most common fluids in Thermodynamics are air, moist air, liquid water, steam, combustion fuels (butane, ethane, methane and propane) and refrigerants like R-134a. The properties for these fluids are the building blocks for solving the problems on the exam. You should also have a concept of what these properties mean in the real world. These concepts will help to reality check your answers, instead of blindly following the results of your equations. Hopefully, this helps you to catch any math errors and speeds up your elimination of incorrect answers.

2.1 PRESSURE

Pressure is one of the two most likely properties that you will start off with in a real world situation, because pressure is a thermodynamic property that is easily measured.

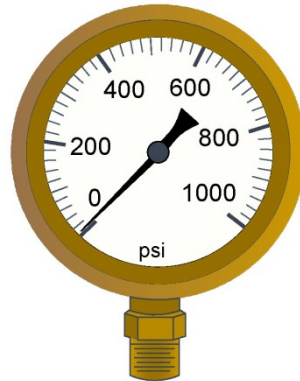


Figure 1: Pressure gauge

The pressure of a fluid indicates the amount of force per unit area that the fluid imparts on the system around it. Pressure is measured in units of pounds per square inch ($psi = \frac{lbf}{square\ inch}$). There are two different types of pressure scales, (1) absolute pressure and (2) gauge pressure. These two pressure scales differ by their 0 reference point. Gauge pressures have a 0-reference point as 1 atm. Thus 0 psig, where the g indicates gauge pressure, is equal to 1 atmospheric or 14.7 psia, where the "a" indicates absolute pressure. Most real world applications encountered by practicing engineers will have pressures indicated in gauge pressure. These include pressures measured at the discharge and intake of pumps and fans and the pressures measured at other pieces of equipment like heat exchangers, chillers and cooling towers. The relationship between gauge and atmospheric pressure is shown with the following equation and figure.

$$P_{abs}[psi] = P_{gauge}[psi] + 14.7\ psi$$

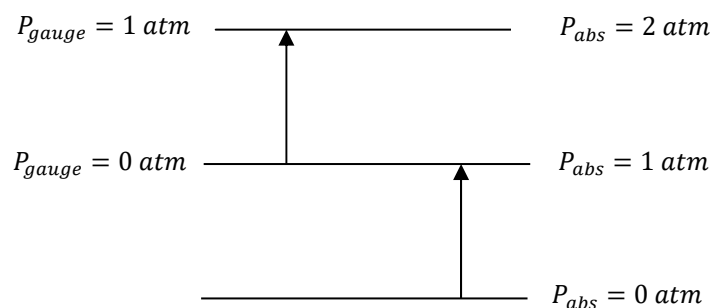


Figure 2: The relationship between gauge and absolute pressures

2.2 TEMPERATURE

Temperature is the second of the two most likely properties that you will start off with in a real world situation, because temperature is easy to measure.

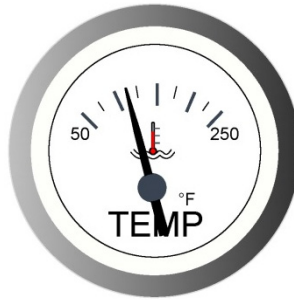


Figure 3: A temperature gauge.

This property is the one most people are familiar with, because it is shown on thermostats and thermometers. Temperature is a direct indication of the amount of heat in the fluid. The USCS units used for temperature are Fahrenheit and Rankine. Typical Fahrenheit temperatures for chilled water (medium used for water-cooled Air Conditioning) range from 45°F to 55°F and hot water temperatures range from 120°F to 140°F. The temperature at which water boils is 212°F and water freezes at 32°F.

Rankine temperatures are used when it is necessary to define an absolute temperature scale having only positive values. The conversion between Fahrenheit and Rankine is shown below. When using equations during the exam, ensure that the correct temperature units are used. Always double check the required units for your equation.

$$^{\circ}R = ^{\circ}F + 460$$

2.3 ENTHALPY

Enthalpy describes the amount of energy in a system. This property is used extensively in HVAC and Thermodynamics. It defines the entering and leaving energy of a fluid through a system. On the exam, enthalpy refers to total enthalpy. Total enthalpy is equal to the internal energy of the fluid plus the energy due to pressure-volume.

$$h = u + pv * \left(\frac{0.18505 \frac{BTU}{lbm}}{1 \text{ psi} * ft^3} \right)$$

$$\text{where } h = \text{enthalpy} \left(\frac{Btu}{lbm} \right); u = \text{internal energy} \left(\frac{Btu}{lbm} \right);$$

$$p = \text{pressure (psi)}; v = \text{specific volume} \left(\frac{ft^3}{lbm} \right)$$

Enthalpy is commonly found in steam tables as shown in the below example.

Temperature	Pressure	Specific volume		Internal energy		Enthalpy		
Temp (°F)	Press. $\left(\frac{lbf}{in^2}\right)$	sat. liquid v_f $\left(\frac{ft^3}{lbm}\right)$	sat. vapor v_g $\left(\frac{ft^3}{lbm}\right)$	sat. liquid u_f $\left(\frac{Btu}{lbm}\right)$	sat. vapor u_g $\left(\frac{Btu}{lbm}\right)$	sat. liquid h_f $\left(\frac{Btu}{lbm}\right)$	evap h_{fg} $\left(\frac{Btu}{lbm}\right)$	sat. vapor h_g $\left(\frac{Btu}{lbm}\right)$
650	2208	.026	0.16	685	1054	696	424	1120

If you take the saturated liquid values and compare them with the above equation, you should find that the equation holds true.

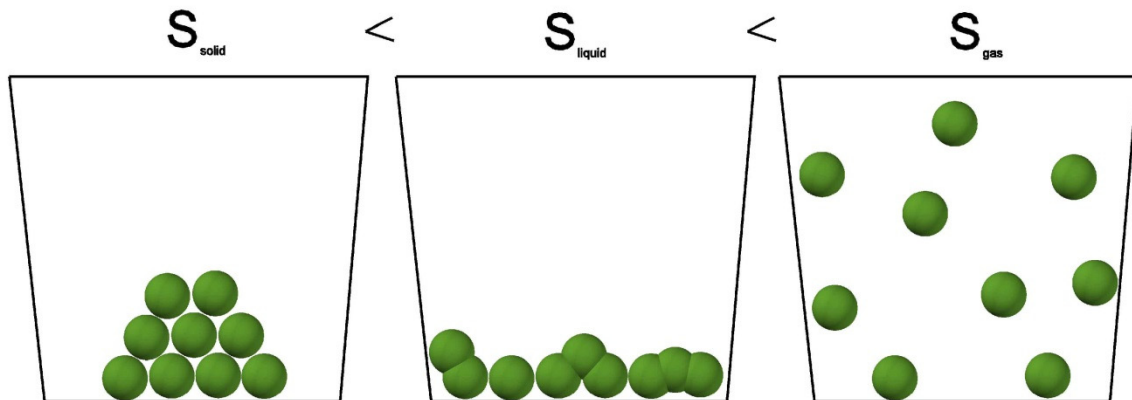
$$h_f = u_f + pv_f * \left(\frac{0.18505 \frac{BTU}{lbm}}{1 \text{ psi} * ft^3}\right)$$

$$696 = 685 + 2,208 * .026 * \left(\frac{0.18505 \frac{BTU}{lbm}}{1 \text{ psi} * ft^3}\right)$$

$$696 = 696$$

2.4 ENTROPY

Entropy is the measure of disorder in a fluid. For example, a solid has low entropy, because it is orderly and the molecules have less possible configurations at any given time. A liquid has a higher entropy than a solid, because of an increased amount of disorder. Finally, a gas has the highest entropy in this example. At any given time, the configuration of the gas can be one of many different configurations.



Entropy is mostly known for its use in the 2nd law, which states that a system's entropy never decreases. Also entropy is used to describe thermodynamic transitions. If there is no change in entropy then the process is determined to be isentropic. Also a process is reversible if the entropy is not increased and the process is irreversible if the entropy increases.

12 – Heat Transfer



Section 13.0 – Heat Transfer

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1.0 INTRODUCTION

Heat Transfer accounts for approximately 7 to 11 questions on the Mechanical FE exam. The heat transfer principles tested on the FE Mechanical exam are shown in the outline below. There are three main areas of heat transfer: conduction, convection and radiation. Conduction is the transfer of heat through contact. In this type of heat transfer, common skills needed include finding overall heat transfer coefficients, finding insulation values and temperature transitions through materials. Convection is the transfer of heat through a moving fluid. This is most commonly seen in heat exchangers as moving hot fluids transfer heat to cool fluids. The main skill needed in this area include finding the convective heat transfer coefficient through the Nusselt number, Reynolds number and/or Rayleigh number. The final type is radiation, which will require finding the radiative heat transfer coefficient and finding the radiative heat transfer between an object and its surroundings.

The last two topics are heat exchangers and transient processes. For heat exchangers, you must be able to calculate the log mean temperature difference for various types of heat exchangers and the heat transfer between the hot and cold fluid.

Section 12.0 Heat Transfer (7 to 11 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Topic 1.0	Introduction
13A	Topic 2.0	Conduction
13B	Topic 3.0	Convection
13C	Topic 4.0	Radiation
13D	Topic 5.0	Transient Processes
13E	Topic 6.0	Heat Exchangers
	Topic 7.0	Boiling
	Topic 8.0	Condensation
	Topic 9.0	Practice Exam Problems
	Topic 10.0	Practice Exam Solutions

2.0 CONDUCTION

Conduction is the method of heat transfer through material(s) in physical contact. The driving force in conduction is a temperature difference on either side of the material(s). For example, if the end of a metal rod is placed in a fire, heat will be conducted through the metal rod to the other end. Heat transfer due to conduction is most commonly calculated for wall and roof heat loads. The outside of a wall or roof is heated by the outdoor conditions. Then the heat is conducted from the outside of the wall through the wall material and to the inside of the wall, where the heat is transferred to the space. The formula for calculating heat transfer due to conduction through a material is as follows:

$$Q = \frac{k * A * (T_{hot} - T_{cold})}{t}$$

where Q = quantity of heat transferred $\left[W \text{ or } \frac{Btu}{hr} \right]$
 k = thermal conductivity of material $\left[\frac{W}{m * ^\circ K} \text{ or } \frac{Btu}{hr * ft * ^\circ F} \right]$
 $T_{hot} - T_{cold}$ = temperature difference $[^\circ K \text{ or } ^\circ F]$
 t = thickness of material $[m \text{ or } ft]$
 A = area of heat transfer $[m^2 \text{ or } ft^2]$

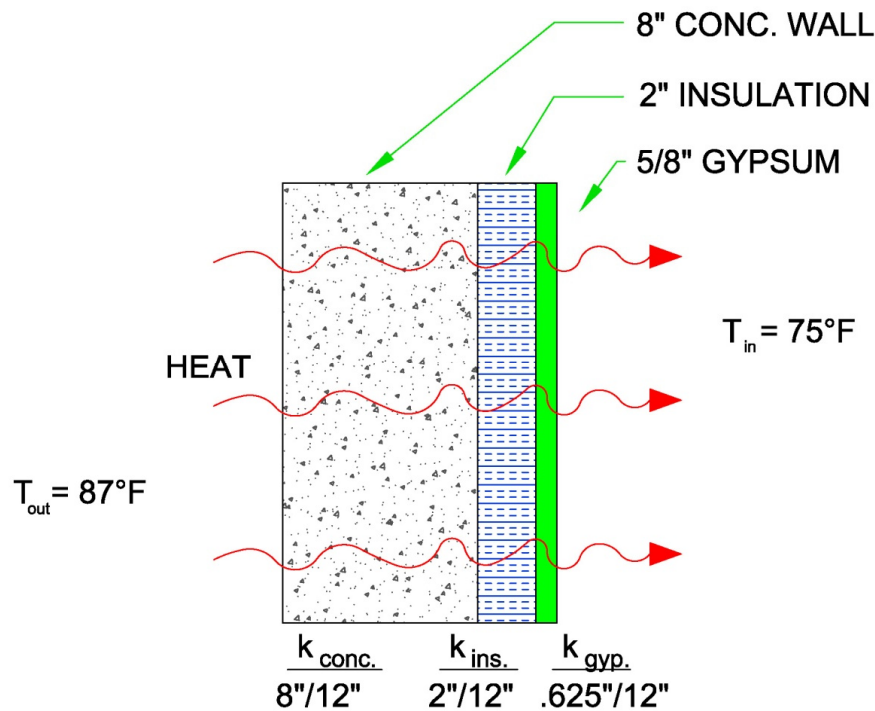


Figure 1: Conduction through a building wall

The amount of heat transferred is linearly dependent on the difference in temperature between the inside and outside surfaces of the wall. The conduction equation shows that as the temperature difference increases, the heat load also increases. The same is also true for the area available for

heat transfer and the thermal conductivity. On the other hand, the amount of heat transferred is inversely related to the thickness of the wall or roof material.

2.1 THERMAL CONDUCTIVITY

Thermal conductivity is a measure of how well a material conducts and promotes heat transfer. Metals are excellent conductors and thus have a high conductivity. For example, aluminum has a thermal conductivity of $128 \frac{\text{Btu}}{\text{hr} * \text{ft} * ^\circ\text{F}}$ and iron has a conductivity of $\sim 30 \frac{\text{Btu}}{\text{hr} * \text{ft} * ^\circ\text{F}}$. Poor conductors include materials like wood (Douglas fir $0.0833 \frac{\text{Btu}}{\text{hr} * \text{ft} * ^\circ\text{F}}$) and insulation materials (Cellular Glass $0.0275 \frac{\text{Btu}}{\text{hr} * \text{ft} * ^\circ\text{F}}$; Glass Fiber $0.0221 \frac{\text{Btu}}{\text{hr} * \text{ft} * ^\circ\text{F}}$).

It is important to note that often times, thermal conductivity is given in units of $\frac{\text{Btu} * \text{in}}{\text{hr} * \text{ft}^2 * ^\circ\text{F}}$. This value is the thermal conductivity per inch thickness of material. Insulation, masonry, plastering and wood materials are often presented with thermal conductivity per inch of materials. As an example, cellular glass has a unit thermal conductivity of $0.33 \frac{\text{Btu} * \text{in}}{\text{hr} * \text{ft}^2 * ^\circ\text{F}}$, which means that for an inch in thickness of cellular glass material, the thermal conductivity is 0.33. For 2" of thickness the thermal conductivity is halved to $0.165 \frac{\text{Btu} * \text{in}}{\text{hr} * \text{ft}^2 * ^\circ\text{F}}$.

$$\text{Thermal Conductivity Units: } \left[\frac{W - \text{mm}}{m^2 * ^\circ K} \right] \text{ or } \left[\frac{\text{Btu} - \text{in}}{\text{hr} * \text{ft}^2 * ^\circ\text{F}} \right]$$

Besides thermal conductivity, materials can also be classified by their R-Value or their U-Factors as shown below.

2.2 U-FACTOR

U-Factor stands for the overall heat transfer coefficient and it is representative of a material's ability to conduct heat. Similar to thermal conductance, a higher U-factor value has a higher ability to conduct and transfer heat. U-factor is related to thermal conductance by the following formula.

$$U = \frac{k}{t} \left[\frac{W}{m^2 * ^\circ K} \right] \text{ or } \left[\frac{\text{Btu}}{\text{hr} * \text{ft}^2 * ^\circ\text{F}} \right]$$

$$Q = U * A * (T_{\text{hot}} - T_{\text{cold}})$$

The units of thickness (t) may be in inches, millimeters, feet or meters. You should pay attention to the units, in order to ensure you use the correct values.

This equation assumes that U does not vary based on temperature. For purposes of the exam, this is a safe assumption.

13 – Measurement, Instrumentations & Controls



Section 13.0 – Measurement, Instrumentation & Controls

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1.0 INTRODUCTION

Measurement, Instrumentation and Controls accounts for approximately 5 to 8 questions on the Mechanical FE exam. This section covers the following topics, sensors, block diagrams, system response and measurement uncertainty. In the sensors topic, you must be familiar with the types of sensors that are used to measure, strain, temperature and pressure, since these are the properties that are most commonly used in mechanical engineering, unlike the pH and chemical sensors which are also shown in the *NCEES FE Reference Handbook*. Block diagrams are used to analyze a control system that consists of different functions in graphical format. You must be able to read and simplify these block diagrams for the FE exam. The system response topic focuses on how control systems respond to various inputs like step, ramp and parabolic inputs. This topic will teach you how to determine if a control system will be stable with the Routh test and how to determine the response error.

Section 13.0 Measurement, Instrumentation and Controls (5 to 8 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
13A	Section 2.0	Sensors and transducers
13B	Section 3.0	Control systems (e.g., feedback, block diagrams)
13C	Section 4.0	Dynamic system response
13D	Section 5.0	Measurement Uncertainty (e.g., error propagation, accuracy, precision, significant figures)
	Section 6.0	Practice Exam Problems

2.0 SENSORS & TRANSDUCERS

The sensors that you need to know for the FE exam are those sensors that measure the difference or change in a specific variable, like temperature. For example, a thermocouple responds to changes in temperature by expanding or contracting. A transducer converts that expansion or contraction into another format like degrees. Typically, the physical measurement, which is measured by the sensor will change a circuit's resistance, which the transducer will use to measure changes in voltage, if assuming that current remains constant.



For the FE exam, you just need to focus on the sensors and transducers used for measuring temperature, strain and pressure because these measurements are shown in the FE Handbook.

2.1 TEMPERATURE SENSORS & TRANSDUCERS

There are two main types of temperature sensors that you need to know for the FE exam, (1) Thermocouple and (2) Resistance Temperature Detector.

The thermocouple uses a composite of two dissimilar metals that creates a voltage as a function of temperature. As the temperature increases, the thermoelectric effect occurs and this effect creates a voltage difference between the two sides of the composite. There are wires that are connected to the opposite sides of the composite that measure the voltage. The voltage increases as the temperature of the composite temperature increases.

The resistance temperature detector works off the basic concept that as a metal increases in temperature its resistance decreases. Thus if the current is maintained constant, then the voltage drop through a metal will decrease as the resistance increases, which increases when the temperature decreases. When the temperature increases, the resistance decreases, which decreases the voltage drop.

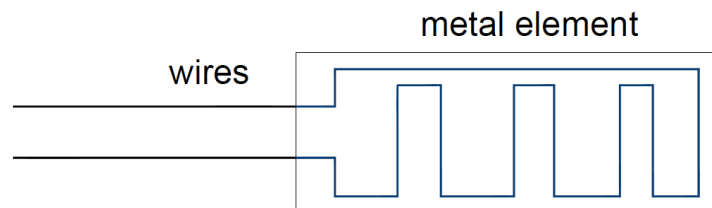


Figure 1: In a resistance temperature sensor, the metal element becomes more conductive as its temperature increases. This reduces the resistance which decreases the voltage drop.

2.2 STRAIN GAUGE

A strain gauge is similar to the resistance temperature detector. A strain gauge consists of a foil-like element with a circuit running through it. This element is placed on a component that will undergo strain. As the component lengthens, the strain gauge will lengthen as well. This will cause the circuit wires within the strain gauge to become narrow, which will increase the resistance of the strain gauge.



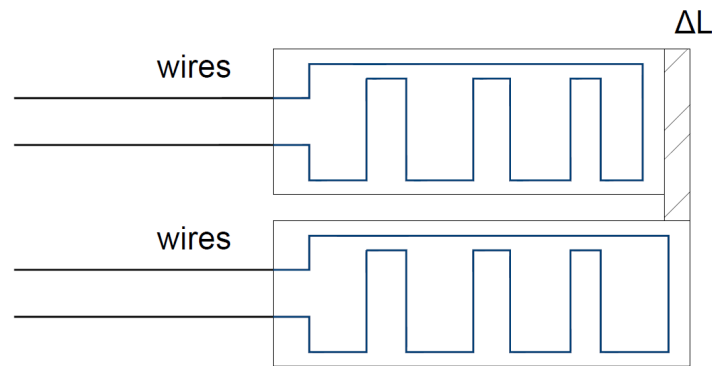


Figure 2: A strain gauge will measure the change in length of a component by measuring the change in resistance of the wires within the strain gauge.

The wires are connected to a Wheatstone bridge to measure the change in resistance. The change in resistance will correspond to a change in length of the component. This change in length can then be used to calculate strain. The ratio between the change in length of the strain gauge and the measured change in resistance is shown as the gauge factor. This gauge factor is pre-measured for each type of strain gauge. Typically the gauge factor is around 2.

$$\text{Gauge Factor} = G = \frac{\Delta R / R_0}{\Delta L / L_0} = \frac{\Delta R / R_0}{\varepsilon}$$

$\varepsilon = \text{strain}$; $\Delta R = \text{change in resistance } (\Omega)$; $R_0 = \text{original resistance } (\Omega)$

$\Delta L = \text{change in length } (m)$; $R_0 = \text{original length } (m)$; $\varepsilon = \text{strain}$

The strain can be solved by rearranging the gauge factor equation.

$$\varepsilon = G * \Delta R / R_0$$

Since, the gauge factor is known, you just need to solve for the difference in resistance of the strain gauge. The change in resistance of the strain gauge is indirectly measured through a Wheatstone bridge. The voltage across the Wheatstone bridge is measured as shown in the following diagram. A known voltage is input into the Wheatstone bridge and the output voltage is measured.



14 – Mechanical Design & Analysis



Section 15.0 – Mechanical Design and Analysis

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1.0 INTRODUCTION

Mechanical Design and Analysis accounts for approximately 10 to 15 questions on the Mechanical FE exam. This section primarily focuses on machine design mechanical components like springs, pressure vessels, beams, piping, bearings, power screws and transmissions. In addition, other topics that support machine design mechanical components are discussed in this section, like manufacturability, quality, and reliability. Then this section switches to topics that are common in Thermal & Fluids like hydraulics, pneumatics and electromechanical components. Finally, this section repeats topics like beams, piping, stress analysis, deformation, stiffness that are covered in previous sections. This section will point you to the correct section when these repeat topics are discussed.

A list of the topics are shown in the table below.

Section 15.0 Mechanical Design & Analysis (10 to 15 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
15A	Section 2.0	Stress Analysis of Machine Elements <i>(repeat from previous Section 8.0 Mechanics of Materials)</i>
15B	Section 3.0	Failure Theories and Analysis
15C	Section 4.0	Deformation and Stiffness <i>(repeat from previous Section 8.0 Mechanics of Materials)</i>
15D	Section 5.0	Springs
15E	Section 6.0	Pressure Vessels and piping
15F	Section 7.0	Bearings
15G	Section 8.0	Power Screws
15H	Section 9.0	Power Transmission
15I	Section 10.0	Joining Methods (e.g., welding, adhesives, mechanical fasteners)
15J	Section 11.0	Manufacturability (e.g., limits, fits)
15K	Section 12.0	Quality and Reliability
15L	Section 13.0, 14.0 & 15.0	Components (e.g., hydraulic, pneumatic, electromechanical)
15M	Section 16.0	Engineering drawing interpretations and geometric dimensioning and tolerancing (GD&T)
	Section 17.0	Practice Exam Problems

2.0 STRESS ANALYSIS OF MACHINE ELEMENTS

Stress analysis of machine elements is discussed in *Section 8.0 Mechanics of Materials*, there are many topics within that section that discuss the stress for all possible types of loads.

3.0 FAILURE THEORIES AND ANALYSIS

Failure can mean a component has been completely fractured; permanently distorted or its function has been compromised. In *Section 9.0 Material Properties and Processing*, various strengths of material properties have been presented. Unfortunately, these strengths which can be used to determine the stress levels at which failure will occur apply only to simple loadings like tension, compression or shear that occurs in one axis. In real life situations and for most of the FE exam, the simple loadings can be assumed. However, there may be a couple of questions on the FE exam where there are complex loadings. For these questions you need to use one of the following failure theories, (1) Maximum Normal Stress Theory for Brittle Materials, (2) Mohr's Theory for Brittle Materials, (3) Maximum Shear Stress Theory for Ductile Materials and (4) Distortion Energy Theory for Ductile Materials also known as Von Mises Theory. Before you learn how to use each of the theories, you need to first understand factor of safety.

In failure theories, the term factor of safety is used to describe the ratio of the load at which the object will fail to the load at which the object will be allowed. The allowed loading can also be described as the design loading. For example, if you design a beam for a loading of 1,000 N then you could choose a beam that fails at 2,000 N. This will result in a factor of safety of 2.

$$\text{Factor of safety (F.S.)} = \frac{\text{Failure loading}}{\text{Allowed or Design loading}}$$

$$FS = 2 = \frac{2,000 \text{ N beam failure}}{1,000 \text{ N beam design}}$$

3.1 MAXIMUM NORMAL STRESS THEORY - BRITTLE

The maximum normal stress theory is similar to the methods presented in *Section 9.0 Material Properties and Processing*. The theory is that if the normal stress in tension or compression is greater than the ultimate strength of the material, then failure will occur.

$$\text{Failure Occurs} \rightarrow \sigma_{\text{tensile}} \geq S_{ut}; S_{ut} = \text{ultimate tensile strength}$$

$$\text{Failure Occurs} \rightarrow \sigma_{\text{compression}} \leq -S_{uc}; S_{uc} = \text{ultimate compressive strength}$$

3.2 MOHR THEORY - BRITTLE

Mohr's theory is another theory used to assess when a material will fail. This theory is used when a material does not have similar compressive and tensile yield strengths. Mohr's theory can be best understood through the use of Mohr's Circle. This circle is a representation of 2-d stresses in the tensile/compressive direction and in shear. The circle is used to analyze complex stresses to determine the principle stresses that are applied to a component. The principle stresses can then be compared to the tensile, compressive and shear strengths.

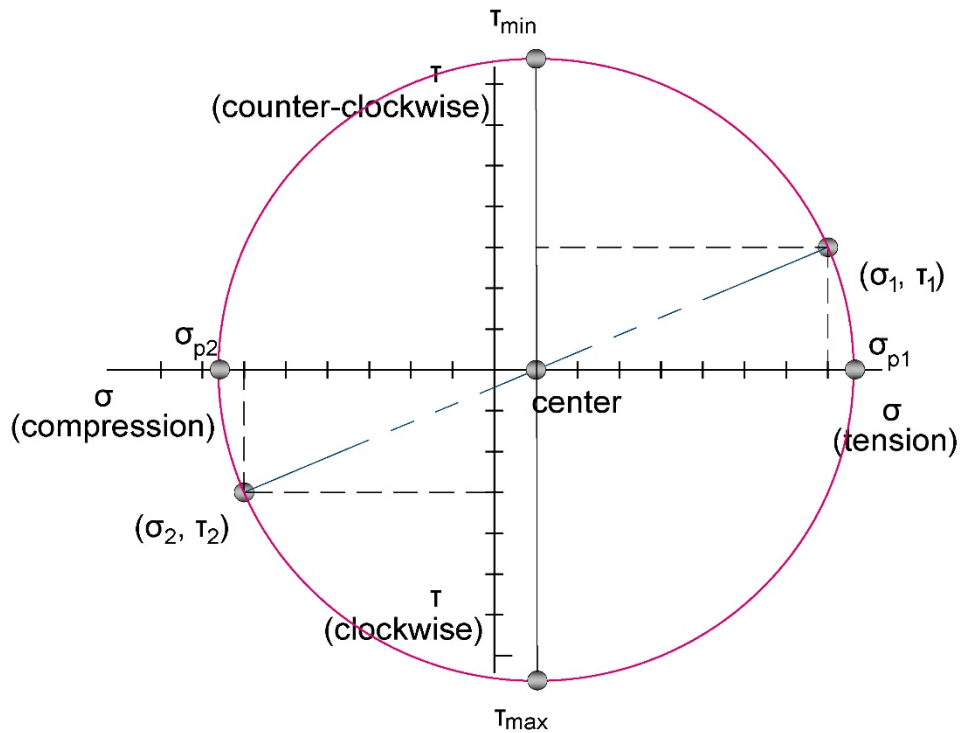


Figure 1: Mohr's circle is used to find the primary stresses in tension, compression and shear.

The complex stresses of (σ_1, τ_1) and (σ_2, τ_2) are typically given in a FE problem. Then you must use Mohr's circle to find the principal forces that act in tension, compression and shear. These principal forces are shown as the extreme points on the circle.

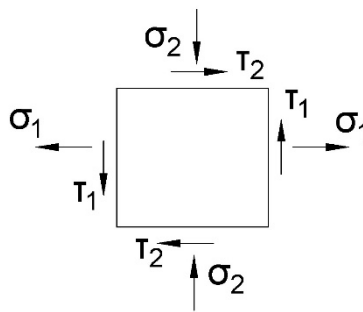


Figure 2: The given stresses that act upon an element within a component are shown in this figure. Notice how the clockwise and counter-clockwise shears correspond to the points shown on the previous figure. Also notice how the tension and compression stresses are shown in this figure correspond to the correct location in the previous figure.

Mohr's circle is created by first determining the center point of the circle from the complex stresses given in the problem. The center is found through this equation.

$$Center = \left(\frac{\sigma_1 + \sigma_2}{2}, \frac{\tau_1 + \tau_2}{2} \right)$$

15 – Checklist Items



	Check List Item	Yes/No?
1	Mathematics	
A	Analytic geometry	
	<i>Can you find the equation of a line, given two points?</i>	
	<i>Can you find the equations of lines that are parallel or perpendicular to another line?</i>	
	<i>Can you find the distance between two points?</i>	
	<i>Are you comfortable with the LOG and LN mathematical properties?</i>	
	<i>Are you comfortable with using LOG and LN functions on your calculator?</i>	
	<i>Are you comfortable with using all of the trig functions and their right angle triangle relationships? SOH - CAH - TOA</i>	
	<i>Can you use the law of sines and cosines?</i>	
	<i>Can you quickly apply trigonometric identities?</i>	
B	Calculus	
	<i>Can you evaluate the derivative for common functions? Find the slope at a certain point on any function.</i>	
	<i>Can you solve for the derivative of various functions?</i>	
	<i>Can you evaluate the integral for common functions? Find the area under a curve.</i>	
	<i>Can you solve for the integral of various functions?</i>	
C	Linear algebra	
	<i>Can you solve for two unknowns with two equations?</i>	
	<i>Can you use matrices to solve multiple equations with multiple unknowns?</i>	
	<i>Can you use the matrices functions on your calculator?</i>	
	<i>Can you solve for matrix determinants, dot products and cross products?</i>	
D	Vector analysis	
	<i>Can you switch between rectangular format and polar format?</i>	
	<i>Can you use your calculator with rectangular values and polar values?</i>	
	<i>Can you conduct vector operations, like addition, multiplication, etc.?</i>	
E	Differential equations	
	<i>Can you solve a first order and second order differential equation?</i>	
	<i>Can you evaluate a first order and second order differential equation?</i>	
F	Numerical methods	
	<i>Can you convert between decimal and binary numbering methods?</i>	



	<i>Can you follow the curve fitting process?</i>	
	<i>Can you follow the least squares process?</i>	
2	Probability & Statistics	
A	Probability distributions	
	<i>Can you calculate mean, mode, and median?</i>	
	<i>Can you calculate standard deviation?</i>	
	<i>Can you calculate the probability of an outcome using the binomial distribution?</i>	
	<i>Can you calculate the probability of an outcome using the normal distribution?</i>	
	<i>Can you calculate the probability of an outcome using the t-distribution?</i>	
	<i>Can you calculate the probability of an outcome using the χ^2-distribution?</i>	
B	Regression & curve fitting	
	<i>Can you follow the linear, square and cubed fit processes?</i>	
	<i>Can you interpret the coefficient of determination?</i>	
	<i>Can you calculate the coefficient of determination for a simple linear fit?</i>	
3	Ethics & Professional Practice	
A	Codes of ethics	
	<i>Did you read the NCEES Model Law and Model Rules</i> https://ncees.org/about/publications/	
B	Agreements and contracts	
	<i>Have you viewed the Engineers Joint Contract Documents Committee knowledgebase? This website is the go to source for engineering contracts.</i> https://www.ejcdc.org/knowledgebase/	
C	Ethical and legal considerations	
	<i>Did you view the National Society of Professional Engineers ethics section?</i> https://www.nspe.org/resources/ethics	
D	Professional liability	
	<i>Have you viewed the Engineers Joint Contract Documents Committee knowledgebase? This website has many liability cases and their results. It gives you an idea on the extent of professional liability and how to protect yourself.</i> https://www.ejcdc.org/knowledgebase/	
E	Public health, safety and welfare	
	<i>Did you read the NCEES Model Law and Model Rules</i>	

16 – Full Exam



1.0 FULL EXAM PROBLEMS

Total time for the 110 questions is 5 hours and 20 minutes.

-- START SESSION 1 --

1.1 PROBLEM 1 – MATHEMATICS

Convert the following value to polar form.

$$5 + \sqrt{-49}$$

- (a) $8.6 \angle 55^\circ$
- (b) $5 + 7 \angle 50^\circ$
- (c) $8.6 \angle -55^\circ$
- (d) $5 \angle -55^\circ$

1.2 PROBLEM 2 – MATHEMATICS

Find the product of the following two vectors, a & b.

$$a = 5 + 6i; \quad b = \sqrt{2} - 7i;$$

- (a) $56 \angle 28^\circ$
- (b) $49 \angle 14^\circ$
- (c) $56 \angle -28^\circ$
- (d) $49 \angle -14^\circ$

1.5 PROBLEM 5 – MATHEMATICS

The following a-b-c triangle has sides and an angle shown below. What is the length of the remaining side?

$$a = 20; B = 75^\circ; c = 30;$$

- (a) 23
- (b) 27
- (c) 31
- (d) 990

1.6 PROBLEM 6 - MATHEMATICS

Find the inflection point of the following equation.

$$y = e^x - 5x^2$$

- (a) (2.3, -16.5)
- (b) (-2.3, -16.5)
- (c) (16, 2)
- (d) (2.3, 0)



1.106 PROBLEM 106 – MECHANICAL DESIGN AND ANALYSIS

An open air tank at sea level feeds a centrifugal pump. The pump's centerline is located 25 feet below the water level of the tank. There is a friction loss of 10 feet head from the tank to the pump suction. The vapor pressure is 0.7 psia. What is the net positive suction head available at the pump suction?

- (a) 8 psi
- (b) 12 psi
- (c) 17 psi
- (d) 21 psi

1.107 PROBLEM 107 – MECHANICAL DESIGN AND ANALYSIS

Which of the following terms is used to characterize compressibility?

- (a) Young's Modulus
- (b) Shear Modulus
- (c) Bulk Modulus
- (d) Rupture Modulus

1.110 PROBLEM 110 – MECHANICAL DESIGN AND ANALYSIS

A bolt will be subject to a shear force of 5,000 N. If the bolt material has a shear strength of 100 MPa, then what is the required bolt diameter?

- (a) .25 mm
- (b) 8 mm
- (c) 17 mm
- (d) 32 mm

2.0 FULL EXAM SOLUTIONS

2.1 SOLUTION 1 – MATHEMATICS

Convert the following value to polar form.

$$5 + \sqrt{-49}$$

Remember that $\sqrt{-1}$ is equal to i

$$5 + \sqrt{-1}\sqrt{49}$$

$$5 + 7i$$

Now convert the complex rectangular number to polar form with your calculator.

$$5 + 7i \rightarrow 2nd \rightarrow complex \rightarrow polar \rightarrow 8.6\angle 54.5^\circ$$

The correct answer is most nearly, **(a) $8.6\angle 55^\circ$** .

- (a) $8.6\angle 55^\circ$
- (b) $5 + 7\angle 50^\circ$
- (c) $8.6\angle -55^\circ$
- (d) $5\angle -55^\circ$

2.2 SOLUTION 2 – MATHEMATICS

Find the product of the following two vectors, a & b.

$$a = 5 + 6i; \quad b = \sqrt{2} - 7i;$$

There are multiple ways to solve this problem, but the easiest is to use your calculator. Make sure you put parentheses around the vectors.

$$(5 + 6i)(\sqrt{2} - 7i) = 49 - 26.5i$$

$$(49 - 26.5i) \rightarrow 2nd \rightarrow complex \rightarrow polar \rightarrow 56\angle -28^\circ$$

The correct answer is most nearly, **(c) $56\angle -28^\circ$** .

- (a) $56\angle 28^\circ$
- (b) $49\angle 14^\circ$



(c) $56\angle - 28^\circ$

(d) $49\angle - 14^\circ$

2.3 SOLUTION 3 – MATHEMATICS

Find the product of the following two vectors, a & b.

$$a = 5 + 6i; \quad b = \sqrt{2} - 7i;$$

There are multiple ways to solve this problem, but the easiest is to use your calculator. Make sure you put parentheses around the vectors.

$$(5 + 6i)(\sqrt{2} - 7i) = 49 - 26.5i$$

The correct answer is most nearly, (d) $49 - 26.5i$.

(a) $7 - 42i$

(b) $56\angle 28^\circ$

(c) $7 + 42i$

(d) $49 - 26.5i$

2.4 SOLUTION 4 – MATHEMATICS

Which of the following is not a solution to the below equation?

$$x^5 + 32 = 0$$

The easiest solution is shown below.

$$x^5 = -32; \quad x = (-32)^{\frac{1}{5}}; \quad x = -2;$$

The next solutions are dependent on the polar coordinates. The first solution can be rewritten in polar coordinates.

$$x = 2\angle 180^\circ;$$

$$x^5 = 2^5\angle(180 + 180 + 180 + 180 + 180)^\circ = 32\angle(900^\circ) = 32\angle(180^\circ)$$

The other solutions must have a magnitude of 2, but the angle must equal 180 degrees or a multiple of 180 degrees, after being multiplied by 5.

$$x = 2\angle 36^\circ;$$



$$x^5 = (2\angle 36^\circ)(2\angle 36^\circ)(2\angle 36^\circ)(2\angle 36^\circ)(2\angle 36^\circ) = 2^5(36 + 36 + 36 + 36 + 36) = 32\angle 180^\circ$$

$$x = 2\angle 108^\circ;$$

$$x^5 = (2\angle 108^\circ)(2\angle 108^\circ)(2\angle 108^\circ)(2\angle 108^\circ)(2\angle 108^\circ) = 2^5(540^\circ) = 32\angle 180^\circ$$

$$x = 2\angle 252^\circ;$$

$$x^5 = (2\angle 252^\circ)(2\angle 252^\circ)(2\angle 252^\circ)(2\angle 252^\circ)(2\angle 252^\circ) = 2^5(1,260^\circ) = 32\angle 180^\circ$$

$$x = 2\angle 322^\circ;$$

$$x^5 = (2\angle 322^\circ)(2\angle 322^\circ)(2\angle 322^\circ)(2\angle 322^\circ)(2\angle 322^\circ) = 2^5(1610^\circ) = 32\angle 180^\circ$$

The correct answer is most nearly, **(d) 1.9 + 0.62i**.

(a) $-0.62 + 1.9i$

(b) -2

(c) $0.62 + 1.9i$

(d) $1.9 + 0.62i$

2.5 SOLUTION 5 – MATHEMATICS

The following a-b-c triangle has sides and an angle shown below. What is the length of the remaining side?

$$a = 20; B = 75^\circ; c = 30;$$

For this problem, you need the law of cosines, which is shown in the NCEES FE Reference Handbook. Please make sure you are in degrees.

$$b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$b^2 = 20^2 + 30^2 - 2(20)(30)\cos(75) = 989.4$$

$$b = 989.4^{.5} = 31.46$$

The correct answer is most nearly, **(c) 31**.

(a) 23

(b) 27

(c) 31

(d) 990