Power PE Technical Study Guide Errata

This product has been updated to incorporate all changes shown in the comments on the webpage and email comments as of June 1, 2020. If you have purchased this product prior to this date and wish for the latest version then please email Justin Kauwale at contact@engproguides.com.

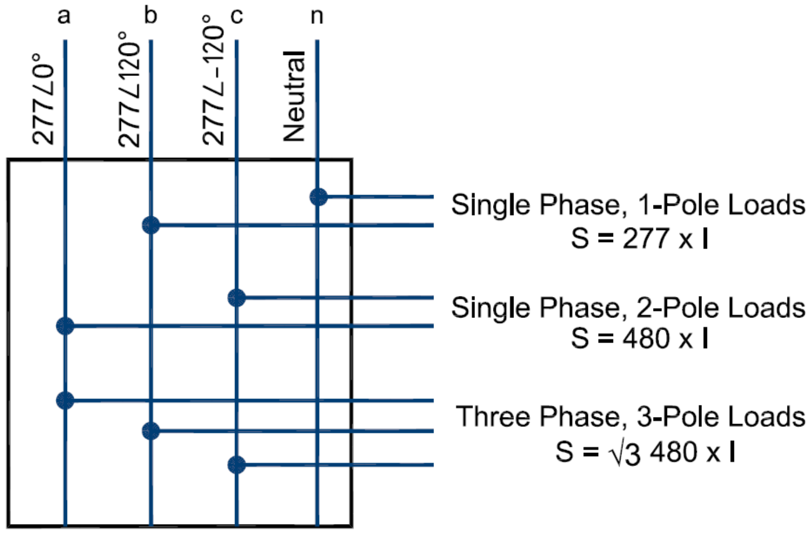


Figure 65: This figure shows the three types of loads. A single pole load has an apparent power equal to 277 V multiplied by the current. A single phase, 2-pole load has an apparent power equal to 480 V multiplied by the current. Lastly, a three-phase, 3-pole load has an apparent power equal to the root 3 factor multiplied by 480 V and the current.

3.9.3 208/120 V Mixed Commercial & Residential (Single and/or Three Phase)

The next power scheme is a 3-phase, 208/120 Volt scheme. In this arrangement, three phases from the utility are sent to a typically delta-wye transformer. The secondary side of the transformer is stepped down to 208 V (phase to phase). If a single phase is referenced to neutral, then the voltage is 120 V. These voltages are typical for mixed residential/commercial buildings.

$$Q_1 + Q_2 = 3.71\ KVAR$$

The power factor can be found with the following equation.

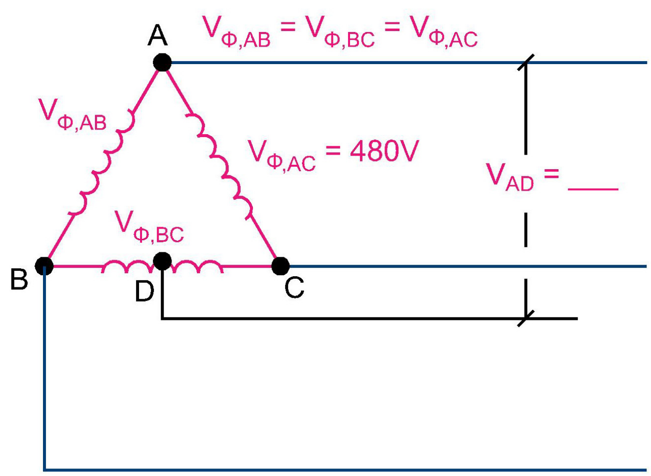
$$\theta = tan^{-1}\left(\frac{3.71}{13.25}\right) = 15.6°$$

$$PF = \cos(15.6°) = 0.96$$

The correct answer is most nearly (d) 0.96.

8.4 Solution 4 - Three-Phase Circuits

A three-phase, delta power source provides power to a three-phase, wye load (3-wire). Assume that a tap is placed on the center of phase C, what is the voltage at the location shown in the diagram?



This configuration is known as a split phase delta.

In this configuration, the voltage between B&D and D&C is equal to 240V. You might be tempted to think that the voltage between A & D is equal to 240V + 480V, but you need to remember that the lines are 120 degree apart from one another. Thus, if you center tap one phase you will get ½ the magnitude at a 120 degree difference.

4.5.3 Three Phase Half Wave Rectifier

The third rectifier is the three-phase, half-wave rectifier. The input to the rectifier are three voltages or currents each at the same RMS value and MAX value. For the purposes of these derivations, voltage will be used, but just understand that voltage can be replaced with current.

$$Input \rightarrow V_{AC,RMS} \;\&\; V_{AC,MAX};\, where\ V_{AC,MAX} = \sqrt{2} * V_{AC,RMS}$$

The function that describes the input is also shown below.

$$Ph - A \rightarrow V_{AC,MAX} * \sin(\theta)\,;\, Ph - B \rightarrow V_{AC,MAX} * \sin(\theta - 120)\,;$$

$$Ph - C \rightarrow V_{AC,MAX} * \sin(\theta + 120)$$

<u>Three Phase, Half Wave Rectifier - Finding DC Average Value</u>

In order to find the DC load (average value) of the rectifier, you must take the area under the curve and divide it by the total period length. The trick is that for a three phase rectifier you are taking the area under the curve of a single sine wave from 30 degrees to 150 degrees. This corresponds to 60 degrees around the peak position at 90 degrees. Each of the phases will contribute the same value, so you will have to multiply this one integration by 3.

$$V_{DC\ load} = 3 * \frac{1}{2\pi} \int_{\pi/6}^{5\pi/6} V_{AC,MAX} * \sin(\theta)$$

$$V_{DC\ load} = 3 * \frac{1}{2\pi} [(-V_{AC,MAX} \cos(5\pi/6) + V_{AC,MAX} \cos(\pi/6))]$$

$$V_{DC\ load,} = 3 * \frac{1}{2\pi} (\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}) V_{AC,MAX}$$

$$V_{DC\ load} = \frac{3\sqrt{3}}{2\pi} V_{AC,MAX} = 0.827 V_{AC,MAX}$$

<u>Three Phase, Half Wave Rectifier - Finding DC Average Value with Delay Angle</u>

In order to find the DC load (average value) of the rectifier, you must take the area under the curve and divide it by the total period length, but similar to the previous delay angle derivations, the second factor will be replaced by cosine theta. Theta is the delay from 30 degrees.

$$V_{DC\ load} = \frac{3}{2\pi} (\cos(\theta)) V_{AC,MAX,Line} = \frac{3\sqrt{3}}{2\pi} (\cos(\theta)) V_{AC,MAX,phase}$$

<u>Three Phase, Half Wave Rectifier - Finding DC RMS Value</u>

4.5.4 Three Phase Full Wave Rectifier

The fourth rectifier is the three-phase, full-wave rectifier. The input to the rectifier are three voltages or currents each at the same RMS value and MAX value. For the purposes of these derivations, voltage will be used, but just understand that voltage can be replaced with current.

$$Input \rightarrow V_{AC,RMS} \,\&\, V_{AC,MAX}; where\ V_{AC,MAX} = \sqrt{2} * V_{AC,RMS}$$

The function that describes the input is also shown below.

$$Ph-A \rightarrow V_{AC,MAX} * \sin(\theta)\,; Ph-B \rightarrow V_{AC,MAX} * \sin(\theta - 120)\,;$$

$$Ph-C \rightarrow V_{AC,MAX} * \sin(\theta + 120)$$

<u>Three Phase, Full Wave Rectifier - Finding DC Average Value</u>

In order to find the DC load (average value) of the rectifier, you must take the area under the curve and divide it by the total period length. The trick is that for a three phase rectifier you are taking the area under the curve of a single sine wave from 60 degrees to 120 degrees. This corresponds to 30 degrees around the peak position at 90 degrees. Each of the phases will contribute the same value twice, so you will have to multiply this one integration by 6.

$$V_{DC\ load} = 6 * \frac{1}{2\pi}\int_{\pi/3}^{2\pi/3} V_{AC,MAX} * \sin(\theta)$$

$$V_{DC\ load} = 6 * \frac{1}{2\pi}\left[\left(-V_{AC,MAX}\cos(2\pi/3) + V_{AC,MAX}\cos(\pi/3)\right)\right]$$

$$V_{DC\ load} = 6 * \frac{1}{2\pi}\left(\frac{1}{2} + \frac{1}{2}\right)V_{AC,MAX}$$

$$V_{DC\ load} = \frac{6}{2\pi}V_{AC,MAX} = 0.9549V_{AC,MAX}$$

<u>Three Phase, Full Wave Rectifier - Finding DC Average Value with Delay Angle</u>

In order to find the DC load (average value) of the rectifier, you must take the area under the curve and divide it by the total period length, but similar to the previous delay angle derivations, the second factor will be replaced by cosine theta.

$$V_{DC\ load} = \frac{3}{\pi}(\cos(\theta))V_{AC,MAX,Line} = \frac{3\sqrt{3}}{\pi}(\cos(\theta))V_{AC,MAX,phase}$$

<u>Three Phase, Full Wave Rectifier - Finding DC RMS Value</u>

In order to find the DC RMS of the rectifier, you must take the area under the square of the curve and divide it by the total period length and then take the square root.

6.5 Problem 5 – Variable Frequency Drives

The AC input voltage to a variable speed drive is 440 V RMS. What will be the voltage measured at the DC link (DC RMS) after the rectifier? Assume 3-phase.

(a) 480 V DC

(b) 597 V DC

(c) 622 V DC

(d) 678 V DC

6.6 Problem 6 – Variable Frequency Drives

Which of the following is methods will provide the greatest reduction in total harmonics caused by a standard 6-pulse variable frequency drive?

(a) Installing a 3% line reactor:

(b) Upgrading from a 6-pulse to an 18-pulse VFD

(c) Upgrading from a 6-pulse to a 12-pulse VFD

(d) Installing a DC inductor

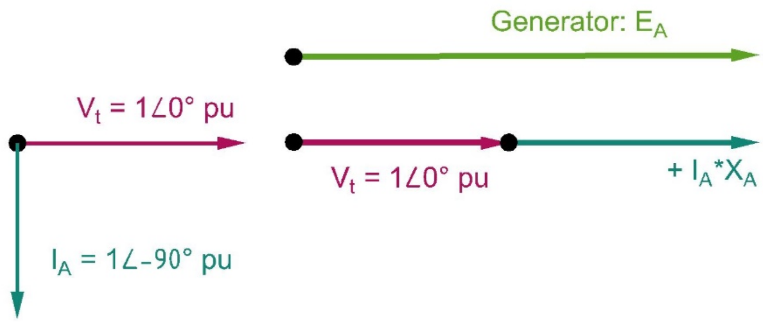


Figure 16: The voltage and current are 90 degrees out of phase. The apparent power is S = I V, so the apparent power at the terminals is completely inductive. The reactance will add an angle of 90 degrees, when the current and voltage are multiplied together, which will cause the resulting voltage drop vector to point horizontal in the positive direction. The Ea vector or what the generator provides must be equal to the terminal voltage plus the voltage drop through the generator equivalent circuit, so the generator Ea vector must be greater than Vt and at the same angle.*

Increase Field Current → Increase Generator Voltage → Q from Generator to System

3. *Leading Load (Load Produces Reactive Power, Q from System to Generator)*
When leading load is added to the system (i.e. reactive power is being delivered from the load to the system), current is leading the terminal voltage by 90 degrees, or $\theta = +90°$. The voltage across the armature winding therefore decreases along the x-axis: $jI_AX_A = [I_A\angle 90°] * [X_A\angle 90°] = I_AX_A\angle 180°$. Then, the internal generator voltage E_A decreases along the x-axis. In summary, when leading load increases, the internal generator voltage must be decreased to maintain the required terminal voltage, and therefore the field current must decrease.

Figure 17: The voltage and current are 90 degrees out of phase. The apparent power is S = I V, so the apparent power at the terminals is completely capacitive. The reactance will add an angle of 90 degrees, when the current and voltage are multiplied together, which will cause the resulting voltage drop vector to point horizontal in the negative direction. The Ea vector or what the generator provides must be equal to the terminal voltage plus the voltage drop through the generator equivalent circuit, so the generator Ea vector must be less than Vt and at the same angle.*

Decrease Field Current → Decrease Generator Voltage → Q from System to Generator

2.3.4.2 Speed Regulation

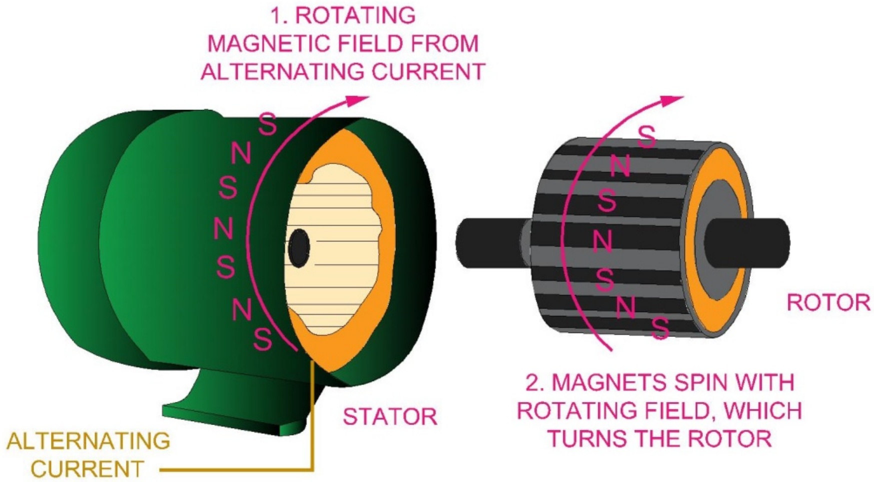


Figure 1: A synchronous machine consists of a stator and a rotor. Motor shown in figure.

2.1.1 Rotating Magnetic Field

A key part of the synchronous and induction machines section is the rotating magnetic field. In a motor, the stator creates a rotating magnetic field. In a generator the rotor creates the rotating magnetic field.

Synchronous Motor: In a motor, alternating current creates a rotating magnetic field in the stator. As one phase becomes reaches its peak it becomes the "North Pole". There is a corresponding "South Pole" that is circuited opposite of that phase's North Pole.

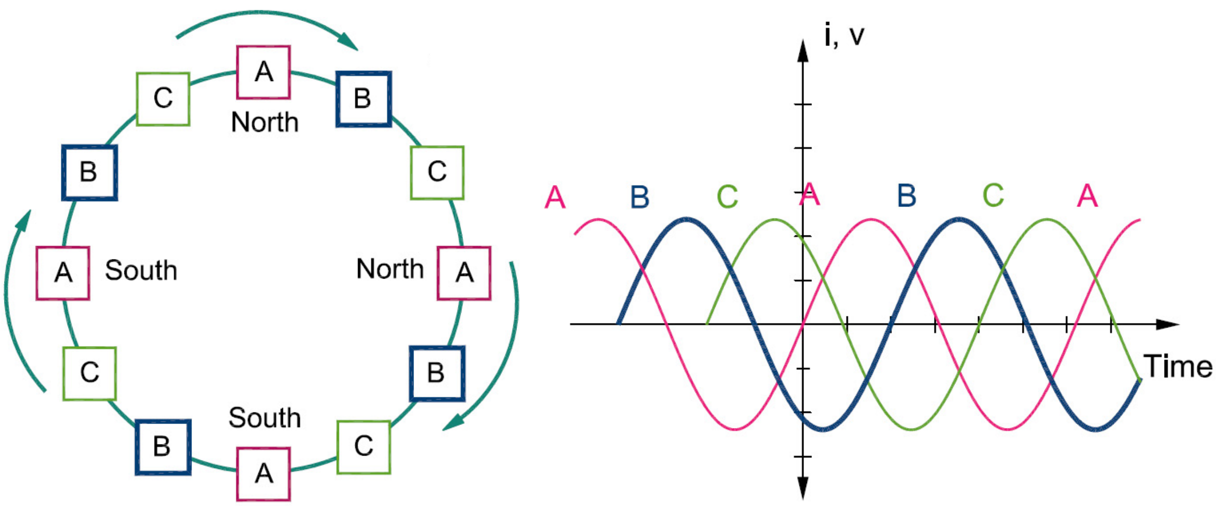


Figure 2: A rotating magnetic field in the clockwise direction, A-C-B sequence.

First, all the "A" slots are either North or South. Then "B" is North or South and then C. This creates a magnetic field rotating clockwise. If the alternating current had a phase sequence of A-

C-B or the slots were arranged differently, then the rotating could be reversed in the counterclockwise direction.

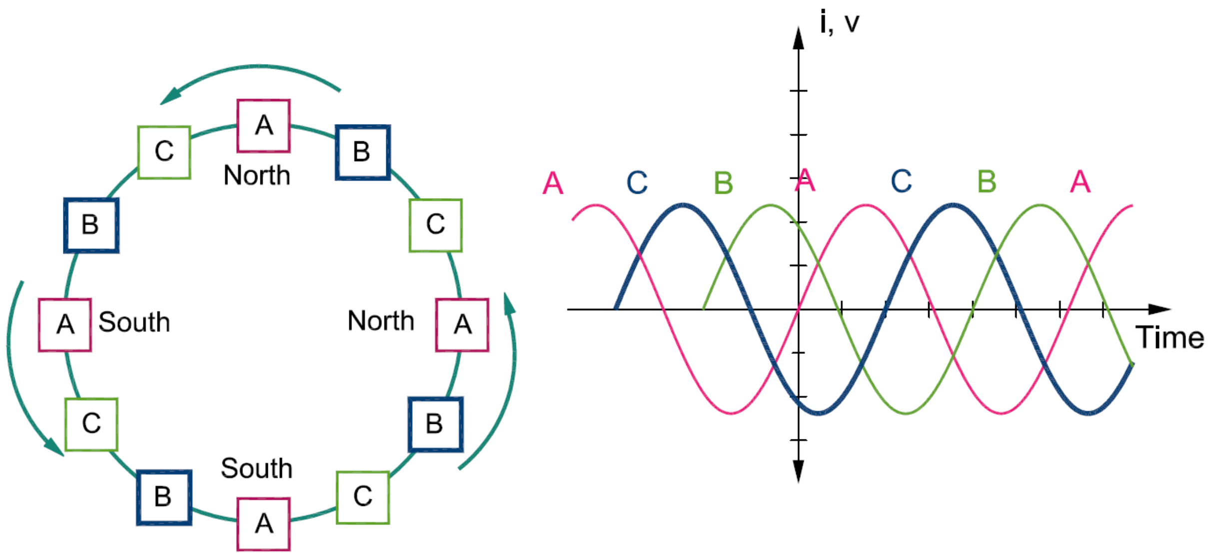


Figure 3: A rotating magnetic field in the counter-clockwise direction, A-B-C sequence

Generator: In a generator, a DC source positively charges and negatively charges poles on the rotor. The generator is connected to a turbine that spins the rotor, which creates a rotating magnetic field.

2.1.2 Torque Angle

After the initial rotating magnetic field is understood, then you must also understand the way the initial rotating magnetic field interacts with the rotor or stator.

Motor: In a motor there is an initial rotating magnetic field in the stator. The rotor will be "jump-started" to rotate at nearly the same speed as the stator's rotating magnetic field. The magnetic fields will then be synchronized or locked together via their magnetic forces. As load is added onto the motor, the rotor will decelerate to create separation between the stator and rotor, but will then accelerate back up to the same speed (synchronous speed). But now there will be a separation angle between the stator and rotor. This is called the torque angle. As the load increases, the torque angle increases.

resistance. The resulting voltage at the air gap is the voltage that the motor will use to rotate the rotor.

On the exam, you should be able to quickly understand the flow of current and voltage drop sequence in the previous equivalent circuit and you should be able to visualize the voltage drop equation in phasor form. The following figures will construct the voltage drop equation in phasor form for a leading power factor load.

2.4.1 Synchronous Motor - Leading Power Factor

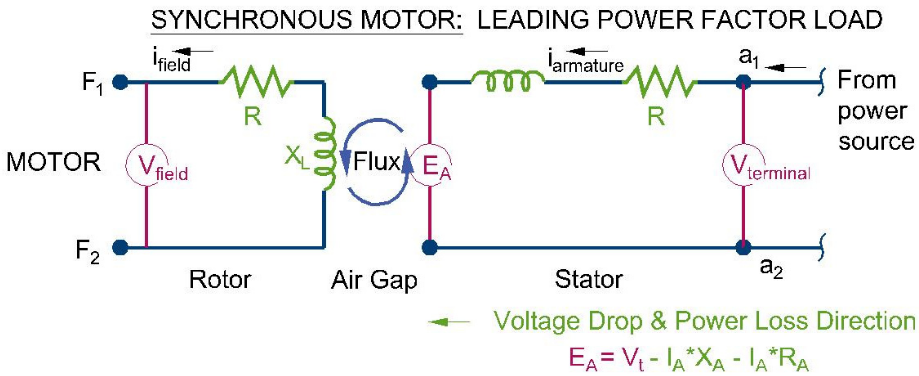


Figure 20: This figure shows the equivalent circuit of a synchronous motor. The circuit is the same for both a leading and lagging power factor, but the following phasor steps are for a leading power factor.

Figure 21: Step 1 - Assign your voltage as having a 0 degree reference point and the armature current leads the voltage phasor by the power factor angle.

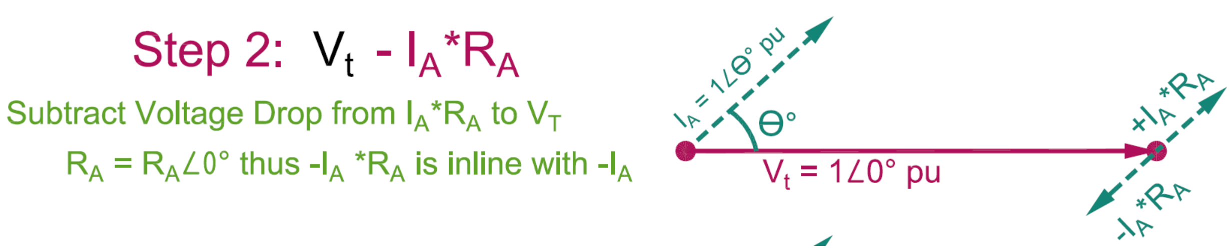


*Figure 22: Step 2 - There will be a voltage **subtraction** from the terminal voltage equal to the voltage drop through the resistive losses. Remember that the current in the equivalent circuit moves from right to left; it starts at the terminal and works its way to the motor. Voltage is being subtracted as you move from the terminal back to the motor. Since, R_A is a pure resistive, its angle is 0 degrees, thus when I_A is multiplied by R_A, an angle of 0 degrees is added to I_A.*

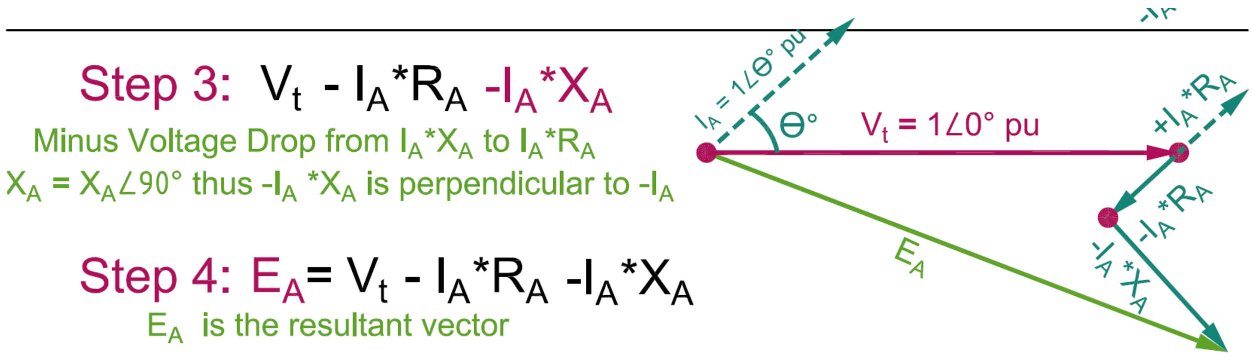
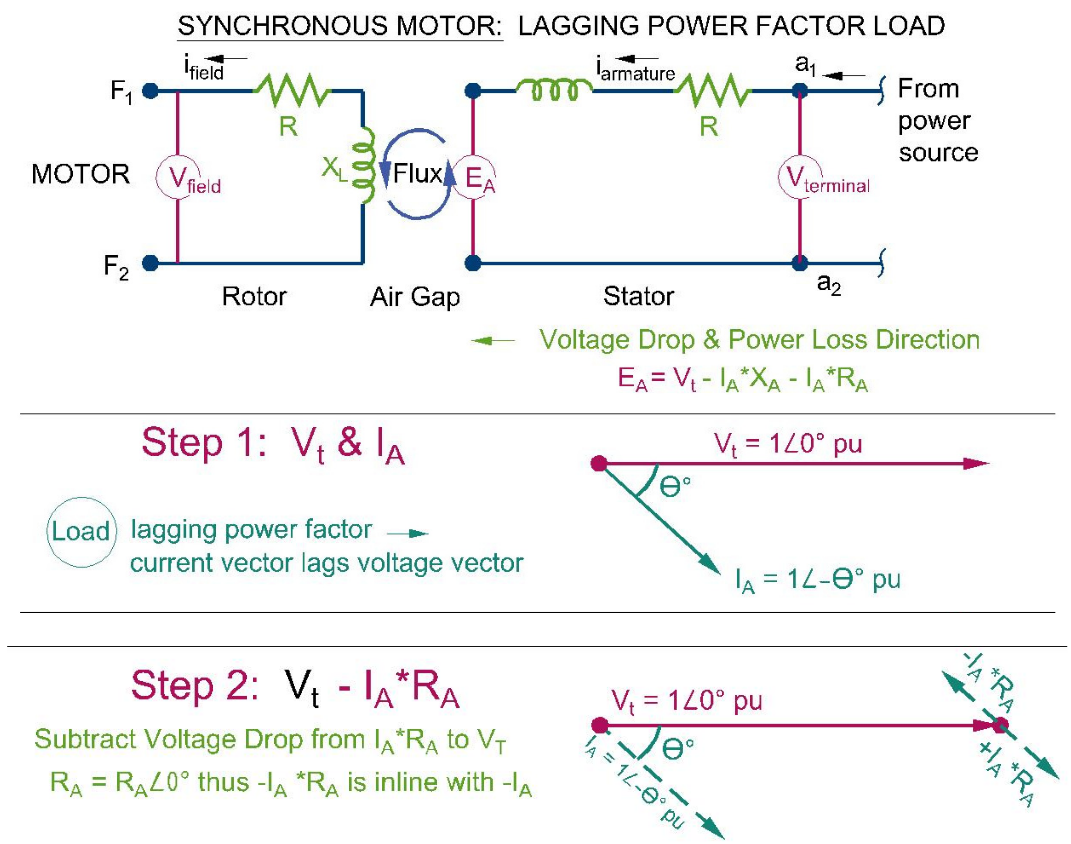


Figure 23: Step 3 - Next, the voltage lost from the inductive losses is subtracted from the terminal voltage phasor. The purely inductive loss, X_A, has an angle of 90 degrees, thus when it is multiplied by I_A, 90 degrees is added to the angle I_A. The resulting vector from the 0,0 point to the final resulting location is E_A.

2.4.2 Synchronous Motor - Lagging Power Factor

The following figure shows the synchronous motor phasor diagram but for a lagging power factor. No step by step description is provided for this figure, but you should be familiar with the differences between the lagging and leading power factor through the phasor diagram, which will be discussed after the figure below.



$$Torque\ (LB-FT) = \frac{Power\ (HP) * 5{,}252}{n_{sync}}$$

The difficulty is finding the real power value. The equivalent circuit for an induction motor is shown below. The real power transferred to the motor is a function of the current and the R/s value at the load.

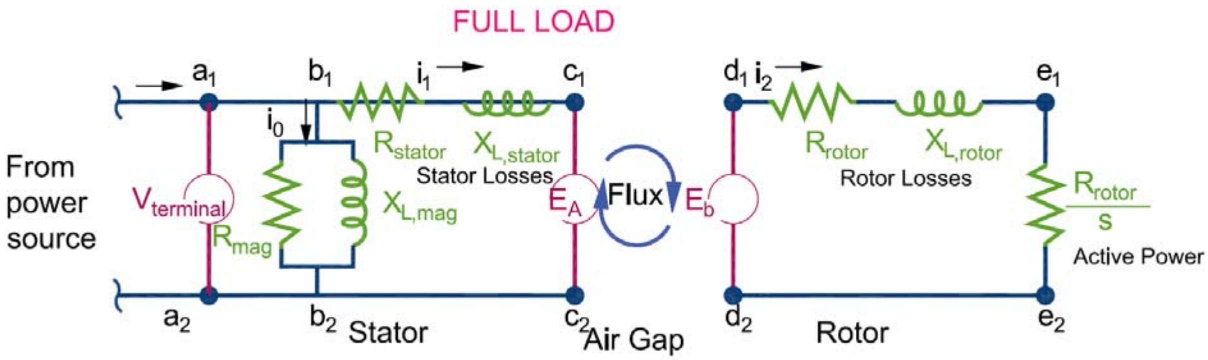


Figure 35: This is a simplified equivalent circuit of an induction motor, there are a few assumptions made in this circuit that are outside the realm of this book. The main things that you need to understand is the flow of power from the terminals of the induction motor to the load. When the induction motor is turned on, there will be a voltage at the terminal end of the circuit. Current will travel through the stator and will build up the stator magnetic field, which will induce a current through the air gap and onto the rotor. The current on the rotor side will also generate a magnetic field, which will oppose the magnetic field and thus voltage at the terminal. Finally, current will enter the load and provide real power.

The equation for real power to the induction motor at the load is shown below.

$$Real\ Power\ = 3 * \frac{I_2^2 * R_{rotor}}{s}$$

The difficulty of calculating the torque and power during start-up conditions is that the current is not constant and is changing. There will be a large inrush of current to initially create the magnetic fields and once these fields are created then current becomes much more stable. So in order to better show the torque and power during start-up, a voltage relationship is created. The equivalent circuit is converted to a Thevenin equivalent circuit. This conversion is very complex and outside of the scope of this book. You just need to understand that the relationship for real power remains the same, but the torque equation is revised as a function of slip and voltage.

$$\sout{Power\ = 3 * \frac{I_2^2 * R_{rotor} * (1-s)}{s}}$$

$$Torque\ = 3 * \frac{R_{rotor}}{s} * \left(\frac{V_{th}^2}{\sqrt{\left(A + \frac{R_{rotor}}{s}\right)^2 + (B+C)^2}}\right)$$

$$P_{motor} = I_2^2 * (R) * \frac{(1-s)}{s}$$

There are multiple factors that affect the active power of the motor. The external resistance is a factor, but it is assumed to remain constant throughout the changes in speed. The slip will be at its highest value when the motor is still at 0 RPM. As the motor gains speed, the slip will slowly decrease. The current will be at its highest value when the motor is just above 0 RPM, this is the largest inrush current, because the resistance to current is at its lowest. The resistance is indirectly related to the slip. When the slip is highest, the resistance is lowest, thus the current is its highest. The power value depends on the multiplication of the square of the armature current and the term, (1-s)/s. This results in a graph similar to the one below.

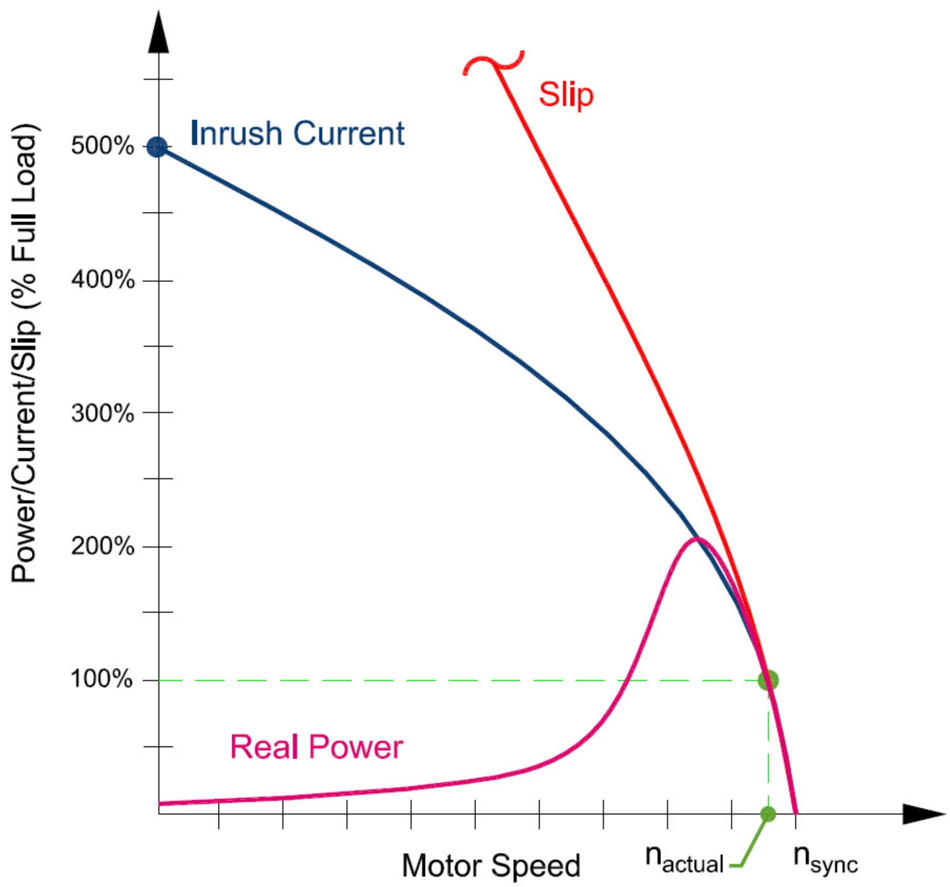
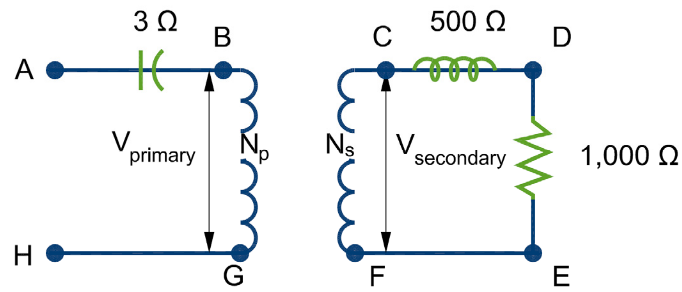


Figure 50: This graph will vary for the specific motor. This is just given to show the relationship between real power, inrush current, slip and the motor speed. At low speeds, the slip is very high, the current is very high and the real power is low. As the speed increases, the slip decreases, the current decreases and the power starts to increase. As the motor continues to accelerate, the current and slip continues to decrease and then the real power starts to decrease. Once the design speed is reached, current, speed and real power should all be at their 100% design values.

The correct answer is most nearly, (B) 1710 RPM. The real power at the design speed, synchronous speed and just at start-up will be much smaller as compared to the speed during the ending of the start-up timeline.



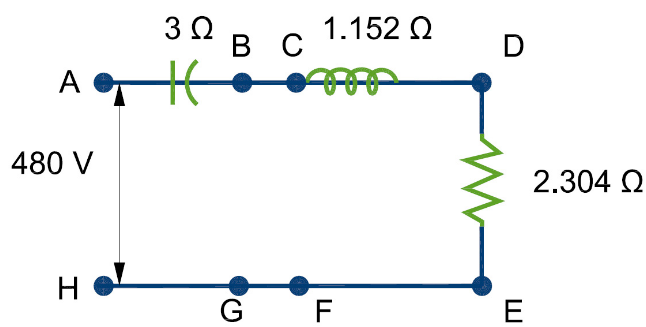
First, transform the secondary impedances to the primary side.

$$\frac{V_{primary}^2}{Z_{primary}} = \frac{V_{secondary}^2}{Z_{secondary}}$$

$$\frac{480^2}{Z_{primary}} = \frac{10{,}000^2}{500\ \Omega} \rightarrow Z_{primary} = 1.152\ \Omega\angle 90°$$

$$\frac{480^2}{Z_{primary}} = \frac{10{,}000^2}{1{,}000\ \Omega} \rightarrow Z_{primary} = 2.304\ \Omega\angle 0°$$

Next, re-draw the circuit but since you only have primary impedances, you only need the primary side.



Now, add up all the impedances, but make sure you use the angles for the capacitor and inductor.

$$Z_{total} = 3\angle -90° + 1.152\ \Omega\angle 90° + 2.304\ \Omega\angle 0° = 2.95\angle -38°$$

Next, solve for the current. You should take the voltage as the zero point.

$$V = 480\ V\angle 0° = I(2.95\angle -38°)$$

$$I = 162.5\angle 38.7°$$

The current is leading the voltage, because the load is primarily capacitive.

Next, multiply the current by the impedance to find the primary voltage drop for each component.

The root number corresponds to the total number of conductors and the quantity under the root symbol is all the possible distances between conductors. This equation results in an equivalent distance between conductors that can be used for the previous equations.

2.1.4 Impedance

On the PE exam, you should be familiar with the typical impedance values for resistance, inductance and capacitance. The resistance impedance values are dependent on the size of the conductor as briefly discussed, thus the resistance impedance values will vary greatly. On the other hand, the inductance and capacitance impedance values do not vary greatly based on conductor size. The values do vary based on the location of the transmission lines. For example, the capacitance of an overhead line is nearly 100 times greater than the capacitance of an underground cable. The inductance of an overhead line is nearly 5 times greater than the inductance of an underground cable. Finally, the impedance due to capacitance is more than 1000 times greater than the impedance due to reactance.

Table 1: Typical impedance values for capacitance and inductance of a transmission line

Line Location	X_L (Ω)	X_C (Ω)
Overhead	0.80/mile	485,000/mile
Underground	0.15/mile	4,850/mile

2.1.5 Short Transmission Line

Now that you understand the various parameters that make up an equivalent circuit, you can put all the terms together to model various transmission lines for analysis. An actual transmission line equivalent circuit will have all three items in its circuit. However for the purposes of the exam and in practice, this equivalent circuit is simplified by removing one or more of the items. The first simplification will be for the short transmission line at low voltage.

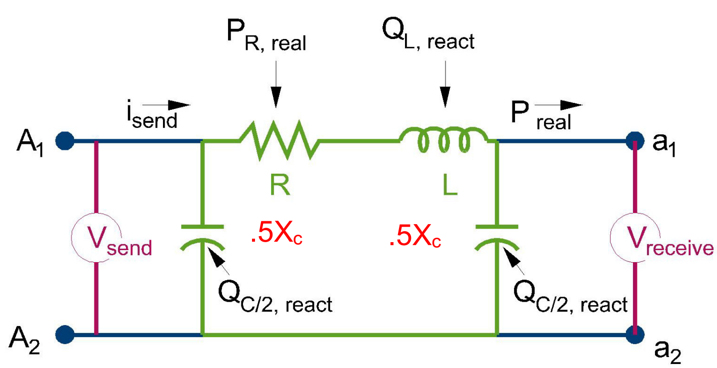


Figure 5: An un-simplified transmission line equivalent circuit.

With the current conjugate, you can find the per phase apparent power by multiplying the conjugate current and the voltage at the desired location.

$$S_{send} = I^{*}V_{send}\angle\theta_2$$

$$S_{send} = j * \frac{(V_{send}\angle-\theta_2 - V_{receive})}{X_A} * V_{send}\angle\theta_2$$

$$S_{send} = j * \frac{(V_{send}\cos(-\theta_2) + jV_{send}\sin(-\theta_2) - V_{receive})}{X_A} * V_{send}\angle\theta_2$$

$$S_{send} = \frac{(jV_{send}\cos(-\theta_2) - V_{send}\sin(-\theta_2) - jV_{receive})}{X_A} * V_{send}(cos\theta_2 + jsin\theta_2)$$

The "j" values refer to the reactive power and the other values refer to the real power.

$$S_{send} = \frac{V_{send}^2\sin(\theta_2)}{X_A}\cos(\theta_2) + \frac{\left(jV_{send}^2\cos(\theta_2) - jV_{receive}V_{send}\right)}{X_A}\cos(\theta_2) + j\frac{V_{send}^2\sin(\theta_2)}{X_A}\sin(\theta_2) + \frac{\left(-V_{send}^2\cos(\theta_2) + V_{receive}V_{send}\right)}{X_A}\sin(\theta_2)$$

$$P_{send} = \frac{V_{receive}V_{send}\sin(\theta_2)}{X_A}; Q_{send} = \frac{\left(V_{send}^2 - V_{receive}V_{send}\cos(\theta_2)\right)}{X_A}$$

Now, you can complete the same derivation for the receiving end.

$$S_{receive} = I^{*}V_{receive}$$

$$S_{receive} = j * \frac{(V_{send}\angle-\theta_2 - V_{receive})}{X_A} * V_{receive}$$

$$S_{receive} = j * \frac{(V_{send}\cos(-\theta_2) + jV_{send}\sin(-\theta_2) - V_{receive})}{X_A} * V_{receive}$$

$$S_{receive} = \frac{(jV_{send}\cos(-\theta_2) - V_{send}\sin(-\theta_2) - jV_{receive})}{X_A} * V_{receive}$$

The "j" values refer to the reactive power and the other values refer to the real power.

$$S_{receive} = -\frac{V_{send}V_{receive}\sin(-\theta_2)}{X_A} + \frac{\left(jV_{send}V_{receive}\cos(-\theta_2) - jV_{receive}^2\right)}{X_A}$$

$$P_{receive} = \frac{(V_{send} * V_{receive})}{X_A} * \sin(\theta_2); Q_{receive} = \frac{\left(V_{send}V_{receive}\cos(\theta_2) - V_{receive}^2\right)}{X_A}$$

$$I_0 = -I_1 * \frac{Z_2}{Z_0 + Z_2}$$

Next, use the symmetrical components to find the phase currents.

$$I_A = I_0 + a^2 * I_1 + a * I_2$$

$$I_B = I_0 + a * I_1 + a^2 * I_2$$

$$I_C = 0$$

3.1.3 Fault Current Examples

One of the best ways to understand fault current analysis is to follow an example calculation. This section will calculate all the fault current and voltages for each of the different types of faults for the figure below.

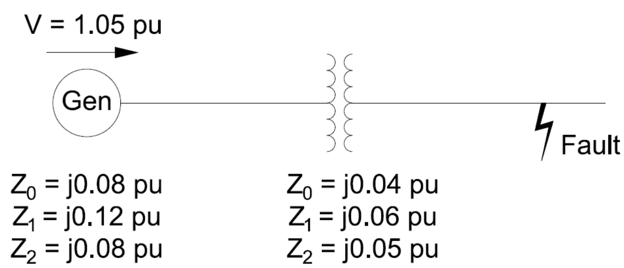


Figure 28: This figure has a generator with a per unit voltage of 1.05. There is also a wye-grounded: wye grounded transformer and the fault occurs on the secondary side of the transformer. Assume that all the per unit impedance values are on the same base. If the impedances are not on the same base, then you would have to convert each impedance to a common base, with the voltage and apparent power values of each component.

For these calculations, assume three phases, A, B & C. Also assume, an A-B-C sequence.

<u>Three Phase Fault:</u>

In a three phase fault, only the positive sequence impedances are included in the calculation. Also the currents will be balanced.

$$I_A + I_B + I_C = 0$$

At the location of the fault, all the voltages will also be equal to zero.

$$V_A = V_B = V_C = 0$$

As an example, the 80% reach will be used to find the maximum load. The load impedance is graphed along with the mho circle, which was created by the 80% line impedance. The impedance can be found as 5.79 by using the angle difference between the line and load impedance. Then you can use the cosine law to find the length from the origin to the intersection of the line impedance line and the circle.

$$\theta_{\Delta} = 75 - 31.8 = 43.6°;\ \cos(43.6) = \frac{X_{load}}{8} \rightarrow X_{load} = 5.79\ \Omega$$

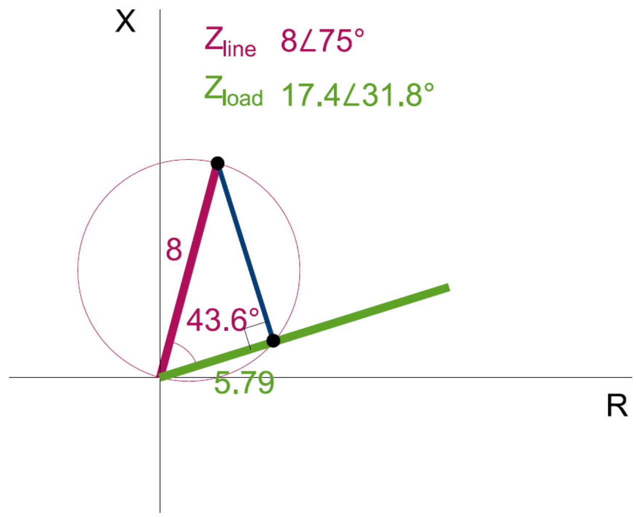


Figure 17: This shows how the angle between the two lines is found. You will need some geometry to find out that the angle between the red and green line is 43.6 degrees and that a right triangle can be formed with the diameter of the circle and the lines connecting the ends of that specific diameter.

The equivalent apparent power can be found with the equation shown below.

$$Magnitude\ only \rightarrow S = \frac{V^2}{Z} = \frac{13{,}200\ V^2}{5.79\ \Omega} = 30\ MVA$$

The normal load is 10 MVA, but if the load were to increase to 30 MVA, then the zone 1 distance relay would trip. This calculation can be completed for various load angles and various line impedance values. The following graph varies the load angle. As you can see from the table, the maximum apparent power is highest for a load angle of 0 degrees. The maximum load is at its lowest value at the same load angle as the line angle.

4.1.7 Distance Relay – Reactance or Resistance Method

A distance relay can also be used to trip only when the resistance or reactance reaches a certain level. The graph for this type of distance relay would consist of a straight vertical or horizontal line, depending on whether or not the relay relied on a resistance or reactance measurement.

7.2 Low Resistance (or Reactance) Grounding

A system that is characterized as low resistance/reactance grounding is directly connected to the ground at any one of its phases or neutral. The figure below shows a ground connection at the neutral point for the wye-side. The delta-side is not grounded. A resistor is placed in the ground connection, which means that during any faults to ground, the short circuit current will be limited by this impedance. On the plus side, any fault can still be detected because of the low resistance. On the negative side, the protection devices and other miscellaneous equipment must be adequately sized for this large current. Although the short circuit current is much less than the solidly grounded system. Another drawback is that there will be a voltage difference due to the resistor, which may create an unsafe condition.

Typically the impedance is selected to limit the current to around 200 to 500 amperes. In the solidly grounded system, the phase to ground fault can be much greater than 500 amperes.

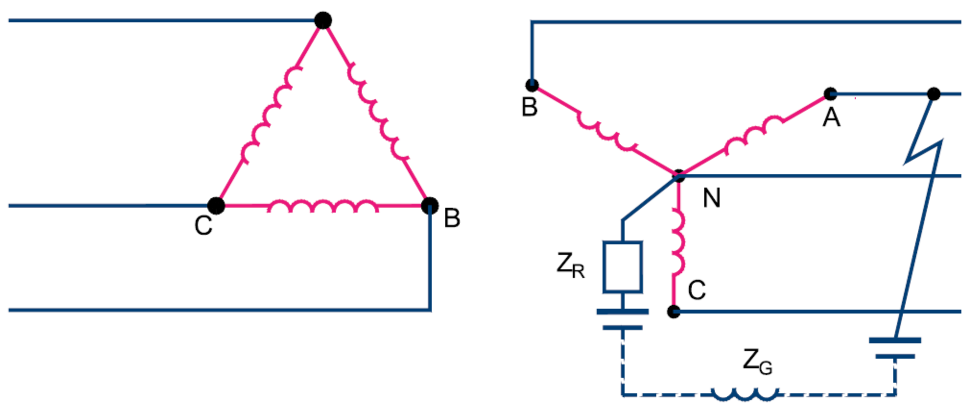


Figure 24: This figure shows a delta-wye transformer. The wye-side of the transformer is solidly grounded. The figure shows a phase to ground fault on phase A, there will be a short circuit connecting A, N, Z_R, and Ground. The Z_G describes any impedance due to ground and Z_R describes the impedance of the intentionally installed resistor/reactor.

The fault current can be calculated for various types of faults as shown below. The 3 phase fault will be the same as for the previous grounding method.

$$I_{SC,3-PH\ Fault} = \frac{V_{fault}}{Z_1}; where\ Z_1 = positive\ sequence\ impedance$$

The phase to ground fault will change, with the addition of the impedance Z_R.

$$I_{SC,PH-G\ Fault} = 3\frac{V_{fault}}{Z_0 + Z_1 + Z_2 + 3Z_R + 3Z_G}$$

But the impedance of the resistor/reactor that is installed is typically much greater than all the other impedances, so the equation can be simplified.

6.0 Practice Problems

6.1 Problem 1 – Short Circuit Current

A 3-phase fault occurs at a location 1. This point is located downstream from a transformer and a conductor. The conductor (3-ph/480V) has an impedance of Z = 2.0 ohms/phase and the transformer has the following properties. 3-phase, 60 Hz, 13.2kV/480V, 10 MVA transformer with Z = 8%. What is the short circuit current at the fault given only the information above?

The answer is most nearly?

(a) 138 A

(b) 240 A

(c) 260,000 A

(d) 450,000 A

6.2 Problem 2 – Differential Relay

A differential relay is used to protect a 3-phase, delta-wye transformer. The transformer is 1,000 MVA, 230/69 KV. The CT ratio on the primary side is 1,000A:5A and the CT on the secondary is 3,000A:5A. What should the magnitude of the differential be, if the fault is calculated as 150% of the rated current? Assume the CTs on the primary side are arranged in wye and the CTs on the secondary side are arranged in delta.

The answer is most nearly?

(a) 8.4 A

(b) 11.6 A

(c) 17.4 A

(d) 21.9 A

7.0 Solutions

7.1 Solution 1 – Short Circuit Current

A 3-phase fault occurs at a location 1. This point is located downstream from a transformer and a conductor. The conductor (3-ph/480V) has an impedance of Z = 2.0 ohms/phase and the transformer has the following properties. 3-phase, 60 Hz, 13.2kV/480V, 10 MVA transformer with Z = 8%. What is the short circuit current at the fault given only the information above?

$$Transformer\ MVA_{SC} = (10\ MVA) * \left(\frac{100\%}{8\%}\right) = 125\ \ MVA$$

$$Transmission\ Line\ MVA_{SC} = \frac{kV^2}{Z} = \frac{0.48^2}{2} = 0.1152\ MVA$$

Next add up the MVA short circuit current values in series.

$$MVA_{SC,total} = \left(\frac{1}{125} + \frac{1}{0.1152}\right)^{-1} = 0.115\ MVA$$

Next use the voltage at the point of fault to find the current.

$$I_{SC} = \frac{0.115 * 1000\ kVA}{0.48\ kV * \sqrt{3}} = 138\ A$$

The correct answer is most nearly, (a) 138 A.

Per Unit Method

$$Z_{base} = \frac{480^2\ V}{10{,}000{,}000\ VA} = 0.02304\ ohms$$

$$Z_{trans,pu} = \frac{2}{0.02304} = 86.8\ pu$$

$$I_{pu} = \frac{1\ pu}{86.8\ pu + 0.08\ pu} = 0.0115\ pu$$

$$I_{actual} = 0.115 * \left(\frac{1{,}000{,}000}{480 * \sqrt{3}}\right) = 138\ A$$

7.2 Solution 2 – Differential Relay

A differential relay is used to protect a 3-phase, delta-wye transformer. The transformer is 1,000 MVA, 230/69 KV. The CT ratio on the primary side is 1,000A:5A and the CT on the secondary is 3,000A:5A. What should the magnitude of the differential be, if the fault is calculated as 150% of the rated current? Assume the CTs on the primary side are arranged in wye and the CTs on the secondary side are arranged in delta.

Remember the rated voltages for equipment are in line voltages. So use the power and line voltage to calculate the line current.

$$S = I * V * \sqrt{3}$$

$$1{,}000{,}000\ KVA = I * 230\ KV * \sqrt{3}$$

$$I_{primary} = 2{,}510\ A$$

$$1{,}000{,}000\ KVA = I * 69\ KV * \sqrt{3}$$

$$I_{secondary} = 8{,}367\ A$$

Next, use the ratios of the CTs to find the transducer CT phase current and then convert to line.

$$I_{primary,ct,ph} = 2{,}510\ A * \left(\frac{5}{1{,}000}\right) = 12.55\ A;\ I_{wye} = 12.55\ A$$

$$I_{secondary,ct} = 8{,}367\ A * \left(\frac{5}{3{,}000}\right) = 13.95\ A;\ I_{delta} = 13.95 * \sqrt{3} = 24.16\ A$$

The magnitude of the difference at 150% rated current is shown through the following equation.

$$= 1.5(24.16A - 12.55\ A)$$

$$= 17.4\ A$$

The answer is most nearly (c) 17.4 A.

7.3 SOLUTION 3 – TCC

A short circuit calculation indicates that a phase to phase fault will cause a short circuit current of 500 A. When will circuit breaker-1 trip, if it has the following characteristics shown in the below graph?

building is difficult to escape, then you are penalized and the value is increased to 3.0. A building that would be difficult to escape would be a high rise or a building with a densely populated occupancy like an auditorium.

Lightning Consequence (C_5): The last factor is the lightning consequence factor. This factor characterizes the importance of keeping this building safe. If the building is hit by lightning and the building's services are not required or will not have any impact on the environment then it gets a factor of 1. If the building is important, then it gets a factor of 5. Lastly, if damage to this building will negatively affect the environment, then it gets a factor of 10.

Finally, the two sides are compared, (1) N_c tolerable lightning frequency and (2) N_d actual lightning frequency.

Actual Lightning Frequency N_D (Describes the building's possible lightning frequency)	Tolerable Lightning Frequency N_C (Describes the building's resistance and avoidance of lightning)
$N_D = (N_G)(Area)(C_D)(10^{-6})$	$N_C = \frac{1.5\ x\ 10^{-3}}{(C_2)(C_3)(C_4)(C_5)}$

$$If\ N_D > N_C, then\ install\ a\ lightning\ protection\ system.$$

$$If\ N_D < N_C, then\ a\ lightning\ protection\ system\ is\ not\ required.$$

2.3 Lightning Protection Systems

The fundamental principle in the protection of life and property against lightning is to provide a means by which a lightning discharge can enter or leave the earth without resulting damage or loss. A low-impedance path that the discharge current will follow in preference to all alternative high-impedance paths offered by building materials such as wood, brick, tile, stone, or concrete should be offered. When lightning follows the higher impedance paths, damage can be caused by the heat and mechanical forces generated during the passage of the discharge. Most metals, being good electrical conductors, are virtually unaffected by either the heat or the mechanical forces if they are of sufficient size to carry the current that can be expected. The metallic path should be continuous from the grounding electrode to the strike termination device. Care should be exercised in the selection of metal conductors to ensure the integrity of the lightning conductor for an extended period. A nonferrous metal such as copper or aluminum will provide, in most atmospheres, a lasting conductor free of the effects of rust or corrosion.

The following sections describe the various grounding methods. You should go through this section and the corresponding grounding section in NEC and understand not only the differences between each method, but the implications of each method on fault calculations.

7.1 Solid Grounding

A system that is solidly grounded is directly connected to the ground at any one of its phases or neutral. The figure below shows a ground connection at the neutral point for the wye-side. The delta-side is not grounded. A solidly grounded system has the lowest resistance to ground, which means that during any faults to ground, there will be a large short circuit current. On the plus side, any fault can be quickly detected because of the low resistance. On the negative side, the protection devices and other miscellaneous equipment must be adequately sized for this large current.

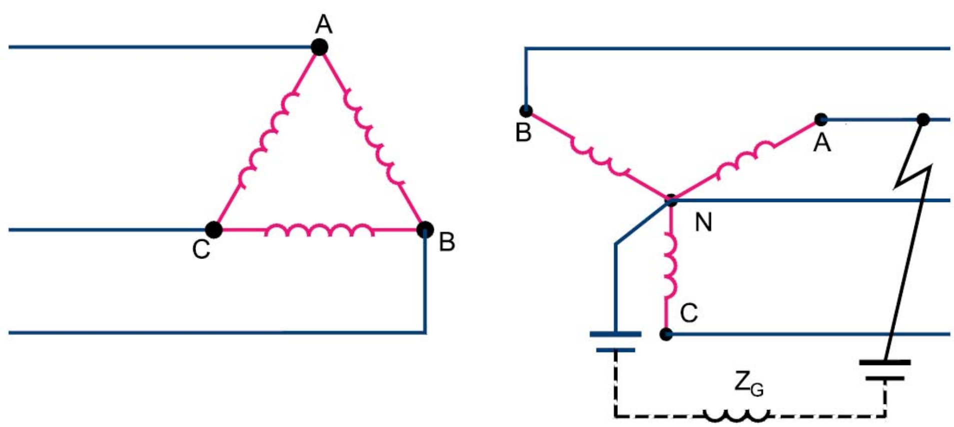


Figure 18: This figure shows a delta-wye transformer. The wye-side of the transformer is solidly grounded. The figure shows a phase to ground fault on phase A, there will be a short circuit connecting A, N and Ground. The Z_G describes any impedance due to ground.

The fault current can be calculated for various types of faults as shown below.

$$I_{SC} = \frac{V_{fault}}{Z_1}; where\ Z_1 = positive\ sequence\ impedance$$

$$I_{SC} = 3\frac{V_{fault}}{Z_0 + Z_1 + Z_2 + 3Z_G}; where\ Z_0\ \&\ Z_2 = zero\ \&\ negative\ sequence\ impedance$$

7.2 Low Resistance (or Reactance) Grounding

A system that is characterized as low resistance/reactance grounding is directly connected to the ground at any one of its phases or neutral. The figure below shows a ground connection at the neutral point for the wye-side. The delta-side is not grounded. A resistor is placed in the ground connection, which means that during any faults to ground, the short circuit current will be

The point 5' away from the wall will result in a point to point distance through the following equation.

$$d^2 = 10^2 + 5^2; \ \theta = \tan^{-1}(5/10) = 26.6°$$

$$Foot - candles_{5\,ft} = \frac{1{,}500\ candela}{10^2 + 5^2\ (ft^2)} \cos(26.6)$$

$$Foot - candles_{5\,ft} = 10.7\ fc$$

The correct answer is most nearly, (b) 12 fc and 15 fc

9.3 Solution 3 - Economics

A client is contemplating on purchasing a new high-efficiency pump and motor, with an initial cost of $10,000. The pump has a lifetime of 15 years and is estimated to save approximately $1,000 per year. There is an additional maintenance cost of $300 per year associated with this new pump. The pump will have a salvage value of $0 at the end of its lifetime. Assume the interest rate is 4%.

Problem: What is the annual value of the pump?

First convert all terms to annual values.

Maintenance cost and cost savings are already annual values.

$$A_{maint} = -\$300$$

$$A_{savings} = \$1{,}000$$

Convert initial cost (present value) to annual value.

$$A_{initial\ cost} = -\$10{,}000 * (\frac{A}{P}, 4\%, 15)$$

Refer to economics tables for value.

$$A_{initial\ cost} = -\$10{,}000 * (.0899)$$

$$A_{total} = A_{maint} + A_{savings} + A_{initial\ cost}$$

$$A_{total} = -\$300 + \$1{,}000 - \$899$$

$$A_{total} = -\$199$$

The correct answer is most nearly, (b) –$199.

Current Rating	Ratio	Secondary Taps
600:5	50:5	X2-X3
	100:5	X1-X2
	150:5	X1-X3
	200:5	X4-X5
	250:5	X3-X4
	300:5	X2-X4
	400:5	X1-X4
	450:5	X3-X5
	500:5	X2-X5
	600:5	X1-X5

Figure 6: The secondary taps column corresponds to a wiring diagram for the current transformer.

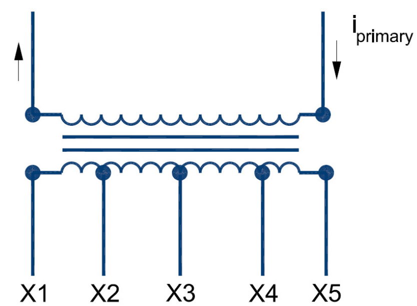


Figure 7: This figure corresponds to the previous table. A combination of two taps would be connected in order to achieve the desired current ratio.

2.1.7 Accuracy Ratings

A current transformer is used to trip relays or to meter current for billing purposes. Thus current transformers need to be accurate. For the exam, you should be familiar with the basics of selecting the accuracy of a current transformer.

Current Ratio	Meter Accuracy	Relay Accuracy	Rating Factor
300:5	0.5 B 2.0	C400	2.0 @ 30 C

The current ratio was previously discussed.

The meter accuracy has three separate terms. The first term, "0.5", describes the percent accuracy of the CT at 2.0 ohms and the rated secondary current. For this example, this means that at 5 amps secondary, the burden will be as calculated below in terms of volt-amperes.

$$Burden = (5\ A)^2 x\ 2\ Ohms = 50\ VA$$

You can also calculate the secondary voltage, assuming the CT is 100% efficient and there are no other losses besides the burden.

$$Secondary\ Voltage = \frac{50\ VA}{5A} = 10\ V$$

Lastly, the accuracy can be assigned to your rated current value for both the primary and secondary.

$$Secondary\ Current = 5\ A\ \pm 0.5\% = 5\ A\ \pm 0.025\ A.$$

$$Primary\ Current = 300\ A\ \pm 0.5\% = 300\ A\ \pm 1.5\ A.$$

The "B" value is the category of current transformer.

Current transformers can also be used for relays as opposed to billing meters. Relays will need an accurate current value for high short circuit current values. The alphabet will either be a "C" or a "T". The "C" stands for calculated, which means you have an ideal current transformer which means that there is very little leakage flux. The "T" stands for tested because you cannot use the ideal current transformer and need to test the current transformer for your actual conditions. The last number is the maximum secondary voltage at 10% accuracy and 20x the rated current. For example, the table shows a value of 400 V and a rated secondary current of 5 A.

$$Maximum\ Secondary\ Voltage = 400\ V = (20\ x\ 5\ A)(X\ ohms)$$

This allows you to calculate the maximum burden in ohms.

$$X = 4\ ohms$$

2.1.8 Rating Factor

The rating factor is the multiple above the rated continuous current that will not cause a significant rise in temperature at the given temperature. The table above indicates that the CT can withstand a continuous 2x of the rated current at an ambient temperature of 30 C. If the ambient temperature is changed, then the square relationship is used. For example, if the temperature was 60 C, then the new rating factor would be as shown below.

$$\left(\frac{New\ Rating\ Factor}{2.0\ Rating\ Factor}\right)^2 = \frac{30\ C}{60\ C}$$

$$New\ Rating\ Factor = 1.41$$

The following figure shows this equation graphically for various rating factors and temperatures. The curves on the figure are the old rating factor and the temperatures are on the X-axis. You will see that the default temperature is 30 C, so for a rating factor of 2.0 at 30 C, the new rating factor will be equal to the old rating factor.

$$Dielectric\ Absorption\ Ratio = \frac{R_{t=60s}}{R_{t=30s}}$$

A good dielectric absorption ratio is above 1.5 and a bad dielectric absorption ratio is below 1.25. In between 1.25 and 1.5 is a decent ratio. In addition to the dielectric absorption ratio, the polarization index is also a good measure of the insulation's resistance.

5.5.2 Polarization Index

The polarization index is the ratio of the resistances at a time of 10 minutes and 1 minute since the voltage was applied.

$$Polarization\ Index = \frac{R_{t=10m}}{R_{t=60s}}$$

A good polarization index is above 4 and a bad polarization index is below 2. In between 2 and 4 is a decent polarization index.

5.6 Megger in Normal Mode

Next, most professional engineers, who use meggers are knowledgeable about the normal and guard mode. These two modes are used for various situations and you need to know when to use each one.

In a normal mode, the megger leads are placed on a single conductor and the ground. This tests the current path from the metal part of the conductor and through the insulation, where it is picked up by the second lead.

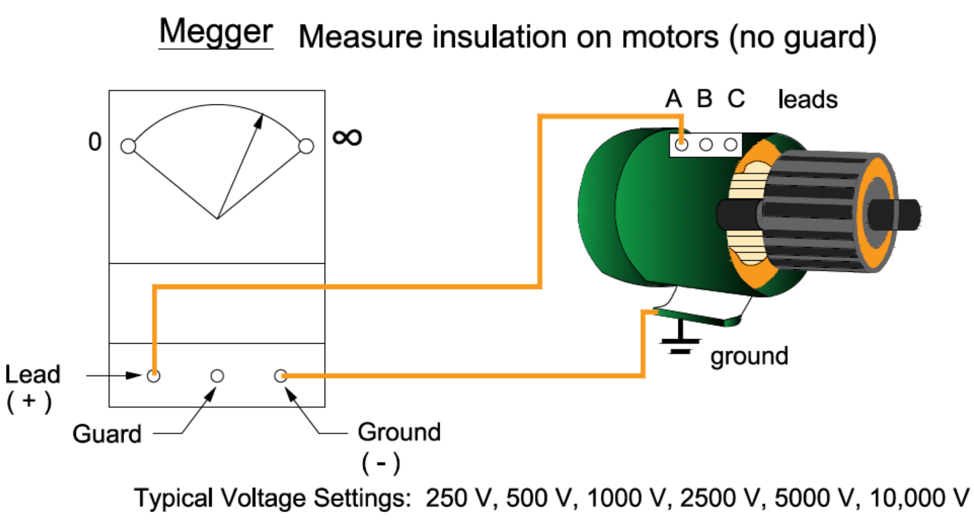


Figure 16: This figure shows the testing of the insulation of conductor A.

You can only use the previous equation when D is sufficiently larger than B (depth of rod), on the order of 10 times B.

$$D > 10B$$

The rod is typically driven 3 to 10 feet into the ground.

6.2 Factors Affecting Measurements

There are several factors that will affect the ground resistance measurements and as a professional engineer, you should know these factors. These factors include ground electrode (rod) diameter, ground rod length, soil type, moisture and temperature.

6.2.1 Ground Rod Length & Diameter

For the exam you should also understand the effects of the grounding rod diameters and depth. The grounding rod diameter provides more surface area with the ground, which will reduce the ground resistance. However, this increase in surface area is not as large as the increase in surface area when the grounding rod is driven to deeper depths. For example, doubling the diameter of the grounding rod will only reduce the resistance by less than 10%. On the other hand, doubling the grounding electrode depth will decrease the resistance by 40%.

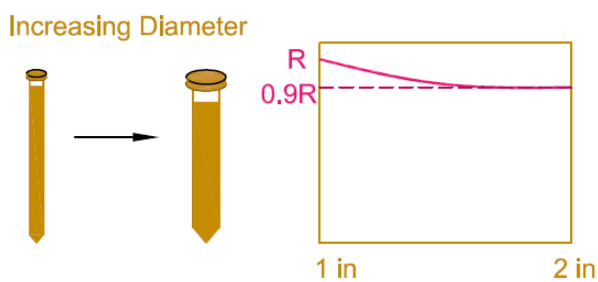


Figure 21: The resistance of the ground is going to be inversely dependent on the surface area of the shells of soil around the ground electrode. As the surface area of those shells increase, the resistance will decrease. As you can see by increasing the diameter, the surface area increases and thus the resistance decreases. As a rule of thumb, doubling the diameter will decrease the resistance by 10%.

7.0 PRACTICE PROBLEMS

7.1 PRACTICE PROBLEM 1 – CURRENT TRANSFORMER

A current transformer with a current ratio of 100:1A is used to monitor a 480V, 1 PH system with a maximum current of 400A. If the CT is reading 2.5A, then what is the current in the system?

(a) 2.5A

(b) 100A

(c) 250A

(d) 400A

7.2 PRACTICE PROBLEM 2 – POTENTIAL TRANSFORMER

A potential transformer with a turns ratio of 100:1 is used to monitor the line to line voltage of the secondary feeds of a 480/120V wye-wye transformer. What is the phase voltage of the transformer secondary when the PT reads 1V? The PT output is arranged in delta.

(a) 1V

(b) 58V

(c) 100V

(d) 120V

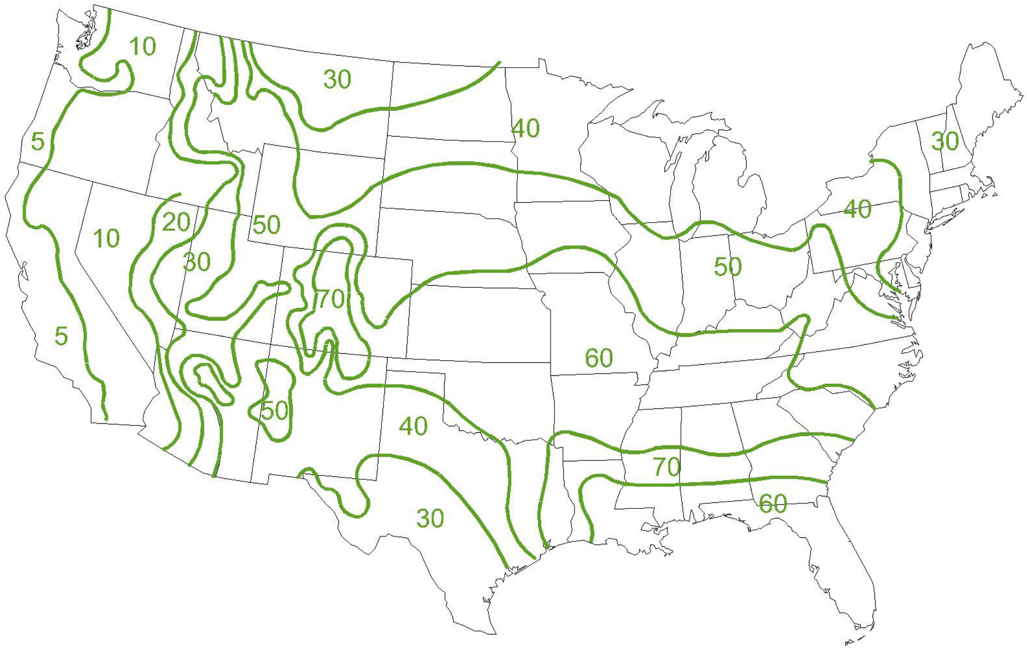


Figure 1: An isokeraunic map shows the relative probability of lightning strikes based on historical data.

This website shows real time lightning strikes in the world. The information gathered from satellites is used to develop the isokeraunic maps. https://www.lightningmaps.org

2.2.1 NFPA 780 Lightning Protection Risk Assessment

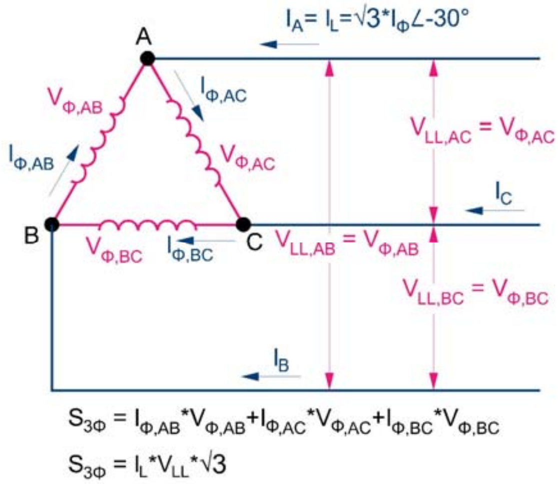
For the PE Exam, you should be familiar with the techniques and designs used to protect buildings and electrical systems (transmission lines, substations, power plants). The *NCEES PE Power Reference Handbook* also indicates that you should be familiar with the NFPA 780 Annex L – Lightning Risk Assessment. The purpose of this assessment is to determine the level of risk associated with a building. Based on the level of risk, you can decide whether or not a lightning protection system is required.

It is important to note that there are two types of assessments, the detailed version and the simplified version. The one presented in the Handbook is the simplified version.

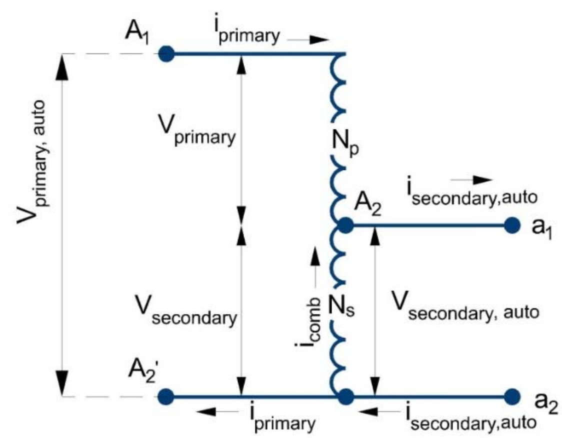
The reference manual takes the risk analysis from NFPA 780 and highlights the main factors that are considered when conducting a Lightning Protection Risk Assessment. The factors that affect the risk analysis include, (1) location factor, (2) building construction, (3) building contents, (4) building occupancy and (5) lightning consequence. The goal of the assessment is to determine the N_c (tolerable lightning frequency) and the N_d (lightning frequency). If the lightning frequency is greater than the tolerable frequency, then a lightning protection system is recommended. If the lightning frequency is less than the tolerable frequency, then a lightning protection system is optional.

Section 2.0 - Circuits Equations

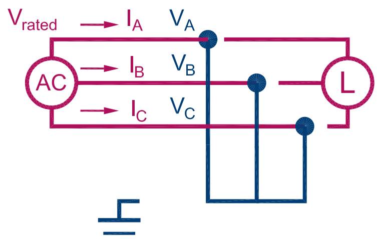
Term	Equation	Description
Symmetrical Components		
"a" term	$a = 1\angle 120°$ $a^2 = 1\angle -120$ $a^3 = 1$	
Positive Component	$I_{A1} = \frac{1}{3}(I_A + a * I_b + a^2 * I_c)$	In terms of the Phases A, B & C. Current can be exchanged for Voltage or Impedance.
Negative Component	$I_{A2} = \frac{1}{3}(I_A + a^2 * I_b + a * I_c)$	
Zero Component	$I_{A0} = I_{B0} = I_{C0} = \frac{1}{3}(I_A + I_b + I_c)$	
Phase A	$I_A = I_{A0} + I_{A1} + I_{A2}$	In terms of the Positive, Negative and Zero Components. Current can be exchanged for Voltage or Impedance.
Phase B	$I_B = I_{A0} + a^2 * I_{A1} + a * I_{A2}$ $I_B = I_{B0} + I_{B1} + I_{B2}$	
Phase C	$I_C = I_{A0} + a * I_1 + a^2 * I_2$ $I_C = I_{C0} + I_{C1} + I_{C2}$	
Line (Phase A) to Ground Fault	$V_A = 0$ $I_B = I_C = 0$ $I_A = I_{A1} + I_{A2} + I_{A0}$ Assume no fault impedance $I_{A1} = I_{A2} = I_{A0} = \frac{E_{A1}}{Z_0 + Z_1 + Z_2}$	
Line (Phase B) to Line (Phase C) Fault	$V_B = V_C$ $I_B = -I_C$ $I_A = 0$ $I_{A0} = 0; I_{A1} = -I_{A2} = \frac{E_A}{jX_1 + jX_2}$ $I_B = I_{A1} * (a^2 - a)$	
Double Line to Ground Fault	$V_B = V_C = 0$ $I_A = 0$ $I_{A1} = \frac{E_{A1}}{Z_1 + (Z_2 \|\| Z_0)}$ $I_{A2} = -I_1 * \frac{Z_0}{Z_0 + Z_2}$ $I_{A0} = -I_1 * \frac{Z_2}{Z_0 + Z_2}$ $I_B = I_{A0} + a^2 * I_{A1} + a * I_{A2}$ $I_C = I_{A0} + a * I_{A1} + a^2 * I_{A2}$	
Three-Phase Fault	$V_A = V_B = V_C$ $I_A + I_B + I_c = 0$ $I_{A1} = \frac{EA_1}{Z_1}$	

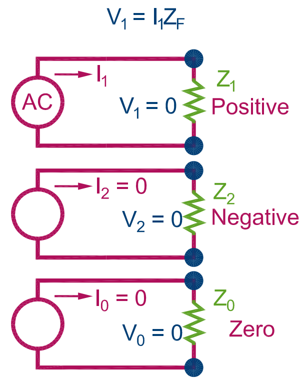
Delta-Connection (add the 30 degree phase shift when going from wye to delta)	Δ $V_L = V_\emptyset$ $I_L = \sqrt{3} I_\emptyset \angle -30°$ $S_{3\emptyset} = 3 V_\emptyset I_\emptyset$ $S_{3\emptyset} = V_L(\sqrt{3} I_L)$ $V_\emptyset = I_\emptyset * Z_\emptyset$ $Z_\emptyset = \frac{3 * V_L^2 \angle -30}{S_{3\emptyset}}$	
Δ → Y Impedance	$Z_\Delta = 3 * Z_Y$	
Δ → Y Voltage	Δ: $V_{ab} = V\angle 0°$ → Y: $V_{an} = \frac{V}{\sqrt{3}} \angle -30°$	
Per-Unit		
Single-Phase		
Per-Unit	$Per\ Unit = \frac{Actual}{Base}$	
Power	$P_{base}, Q_{base}, or\ S_{base} = V_{base} * I_{base}$	
Impedance	$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{(V_{base})^2}{S_{base}}$	
Admittance	$Y_{base} = \frac{I_{base}}{V_{base}} = \frac{1}{Z_{base}}$	
Three-Phase		
3Ø to 1Ø Power Conversion	$S_{base,3\emptyset} = 3 * S_{1\emptyset,base}$	
Current	$I_{\emptyset,base} = \frac{S_{base,3\emptyset}}{3 * V_{\emptyset,base}}$ $I_{L,base} = \frac{S_{base,3\emptyset}}{\sqrt{3} * V_{L,base}}$	
Impedance	$Z_{base} = \frac{3(V_{\emptyset,base})^2}{S_{base,3\emptyset}}$ $= \frac{(V_{LL})^2_{base}}{S_{3\emptyset,base}}$	
Change of Base, Impedance		
General	$Z_{pu,new} = Z_{pu,old} \frac{(S_{new})(V_{old})^2}{(S_{old})(V_{new})^2}$	

Parallel Transformers		
Impedance *NCEES 125*	$S_{1,actual} = S_{total,actual} \dfrac{\frac{S_{1,rated}}{Z_{1\%}}}{\frac{S_{1,rated}}{Z_{1\%}} + \frac{S_{2,rated}}{Z_{2\%}}}$ $S_{2,actual} = S_{total,actual} \dfrac{\frac{S_{2,rated}}{Z_{2\%}}}{\frac{S_{1,rated}}{Z_{1\%}} + \frac{S_{2,rated}}{Z_{2\%}}}$ $S_{total,actual} = S_{1,actual} + S_{2,actual}$	Finding the flow of power between the two transformers through the rated transformer KVA capacities and their % impedance values.
Impedance (per-unit method)	$Z_{2,base\ 1}\% = Z_{2,base\ 2}\% * \dfrac{S_{1,rated}}{S_{2,rated}}$ $S_{1,actual} = S_{total,actual} \dfrac{Z_{2,base\ 1}\%}{Z_{1,base\ 1}\% + Z_{2,base\ 1}\%}$ $S_{2,actual} = S_{total,actual} \dfrac{Z_{1,base\ 1}\%}{Z_{1,base\ 1}\% + Z_{2,base\ 1}\%}$	Converting impedance of Z_2 to the same base as Z_1 when $Z_1 \neq Z_2$. Once you have the % impedances in the same base, then you can use the below equation to find the flow of power through two parallel transformers.
Reactors		
Inductance coils that limit current during fault conditions		

Step-Down Autotransformer		
	$V_{p,auto} = V_p + V_s$ $I_{s,auto} = I_p + I_{comb}$ $\dfrac{N_P}{N_S} = \dfrac{V_P}{V_S} = \dfrac{I_{comb}}{I_P}$ $\dfrac{N_P + N_S}{N_S} = \dfrac{I_{S,auto}}{I_P}$	

3-Phase Bolted Fault





No fault impedance

$$V_A = a^2V_B = aV_C = V_1 = 0$$

$$I_A = a^2I_B = aI_C$$

$$I_A + I_B + I_C = 0$$

$$V_{A2} = V_{A0} = I_{A2} = I_{A0} = 0$$

$$I_{A1} = I_A = \frac{V_{rated}}{Z_1}$$

With fault impedance

$$V_A = (\tfrac{1}{3}) \times I_A \times Z_F$$

$$V_1 = I_1 \times Z_F$$

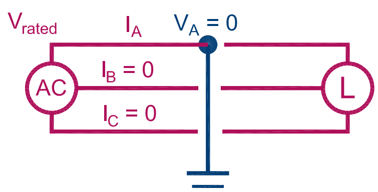
$$I_A + I_B + I_C = 0$$

$$V_{A2} = V_{A0} = I_{A2} = I_{A0} = 0$$

$$I_{A1} = \frac{V_{rated}}{Z_1 + (\tfrac{1}{3})Z_F}$$

$$I_{A2} = I_{A0} = 0$$

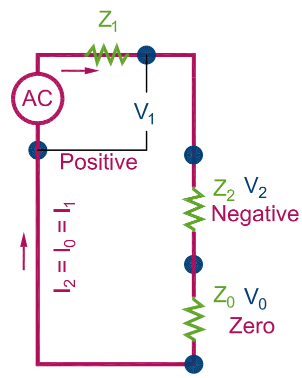
1-Phase to Ground Fault



$V_1 = I_1 * (Z_2 + Z_0 + 3Z_F)$

$V_2 = -I_1 * Z_2$

$V_0 = -I_1 * Z_0$



No fault impedance

$$V_A = 0 = V_0 + V_1 + V_2$$

$$I_B = I_C = 0$$

$$V_B = V_0 + a^2V_1 + aV_2$$

$$V_C = V_0 + aV_1 + a^2V_2$$

$$I_{A1} = I_{A2} = I_{A0} = \frac{V_{rated}}{Z_0 + Z_1 + Z_2}$$

$$I_A = I_{A1} + I_{A2} + I_{A0}$$

$$I_A = 3\frac{V_{rated}}{Z_0 + Z_1 + Z_2}$$

With fault impedance

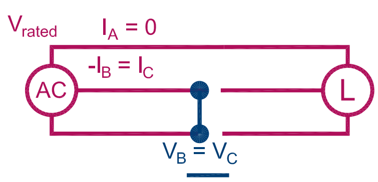
$$V_A = I_A \times Z_F$$

$$V_{rated}$$

$$= \frac{}{Z_0 + Z_1 + Z_2 + 3Z_F}$$

$$= \frac{3 \times V_{rated}}{Z_0 + Z_1 + Z_2 + 3Z_F}$$

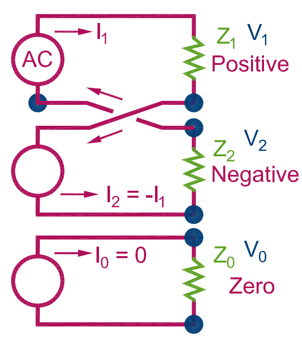
Phase to Phase Fault



$V_1 = I_1 * (Z_2 + Z_F)$

$V_2 = -I_1 * Z_2$

$V_0 = 0$



No fault impedance

$$V_B = V_C = V_1 + V_2 = 0$$

$$I_B = -I_C \quad I_A = 0$$

$$I_0 = 0$$

$$I_1 = -I_2 = \frac{V_{rated}}{Z_1 + Z_2}$$

$$I_A = I_0 + I_1 + I_2 = 0$$

$$I_B = I_0 + a^2I_1 + aI_2$$

$$I_C = I_0 + aI_1 + a^2I_2$$

With fault impedance

$$V_B - V_C = I_B \times Z_F$$

$$V_1 - V_2 = I_1 \times Z_F$$

$$I_0 = 0$$

$$I_1 = -I_2 = \frac{V_{rated}}{Z_1 + Z_2 + Z_F}$$

$$I_A = I_0 + I_1 + I_2 = 0$$

$$I_B = I_0 + a^2I_1 + aI_2$$

$$I_C = I_0 + aI_1 + a^2I_2$$